vertex operator algebras

Delving into the Depths of Vertex Operator Algebras: A Comprehensive Guide

Introduction:

Have you ever encountered mathematical structures so elegant and powerful that they bridge seemingly disparate fields of mathematics and physics? Enter the world of Vertex Operator Algebras (VOAs), fascinating mathematical objects with deep connections to conformal field theory, string theory, and even monstrous moonshine. This comprehensive guide will demystify VOAs, exploring their fundamental concepts, key properties, and significant applications. We'll navigate the intricacies of their definition, examine their representations, and touch upon their profound impact on various areas of research. Prepare for a journey into the heart of a vibrant and rapidly evolving field of mathematics.

1. What are Vertex Operator Algebras? A Foundational Overview:

Vertex Operator Algebras are not easily defined in a single sentence. They are highly structured algebraic objects characterized by a collection of operators (vertex operators) acting on a vector space (often infinite-dimensional). These operators, indexed by elements of a vector space (often the complex numbers), satisfy a complex set of axioms that encode the algebraic properties of these operators. Crucially, these axioms reflect a rich underlying symmetry, often connected to conformal symmetry prevalent in two-dimensional conformal field theory. Think of them as a sophisticated generalization of Lie algebras, capturing a much richer algebraic structure.

The core of a VOA lies in the precise way these vertex operators interact. They obey intricate commutation relations, governed by the Borcherds identities, which serve as the defining axioms of a VOA. These identities capture the intricate interplay between the vertex operators and guarantee a consistent algebraic structure. Understanding these identities is paramount to grasping the essence of VOAs.

1. Key Components and Properties of VOAs:

Several key components define a VOA:

The vector space: This is the underlying space on which the vertex operators act. It's usually infinite-dimensional and carries a significant amount of structure.
The vertex operators: These are the central objects, providing a way to "create" new elements in the vector space by acting on existing ones. They are highly structured operators, encoding the algebraic and geometric information within the VOA.

The vacuum vector: This special vector plays a fundamental role in the axioms. It's typically invariant under the action of the vertex operators and serves as a starting point for constructing other elements in the vector space.
The Virasoro algebra: Many VOAs possess a subalgebra isomorphic to the Virasoro algebra, a crucial Lie algebra connected to conformal symmetry. The central charge, a crucial invariant of the Virasoro algebra, is a key characteristic of the VOA.

Key properties include:

Locality: This property dictates how the vertex operators commute (or rather, how their commutator is expressed), which is crucial for the consistency of the structure.
Associativity: This property relates the compositions of vertex operators in different orders, ensuring the consistency of calculations involving multiple vertex operators.
Translation invariance: This reflects the inherent symmetry within the structure.

1. Representations and Modules of VOAs:

Just as groups have representations, VOAs also possess representations, which are vector spaces on which the VOA acts in a way that respects its algebraic structure. Understanding these representations is vital for analyzing the structure of the VOA and extracting meaningful information. Irreducible representations, those that cannot be further decomposed into smaller representations, are particularly important. The study of these modules reveals a deeper understanding of the structure and properties of the VOA itself.

Different types of representations exist, each carrying its own significance. For example, studying the highest-weight representations of VOAs offers a powerful tool for analyzing their structure. These representations are built upon a "highest-weight vector," a vector annihilated by certain operators in the VOA. The classification and analysis of these representations provide crucial insights into the VOA's underlying algebraic structure.

1. Connections to Conformal Field Theory and String Theory:

VOAs play a pivotal role in conformal field theory (CFT), a framework used to study two-dimensional systems with conformal symmetry. VOAs provide a rigorous mathematical framework for describing the algebraic structure of CFTs. The vertex operators in a VOA correspond to the operators in the CFT that create and annihilate excitations of the quantum field. The axioms of the VOA reflect the symmetry properties of the CFT.

The connection extends to string theory, where CFTs are fundamental building blocks for constructing string models. VOAs provide a powerful tool for analyzing string interactions and constructing consistent string theories. Different VOAs correspond to different string models with diverse properties and physical interpretations.

1. Monstrous Moonshine and its Implications:

One of the most remarkable applications of VOAs involves the celebrated "Monstrous Moonshine" conjecture. This astonishing connection links the monster group, the largest sporadic finite simple group, to the modular functions of the j-invariant. This unexpected connection, initially observed through numerical coincidences, was eventually explained using VOAs. The construction of a specific VOA, the moonshine module, demonstrated a deep and profound connection between seemingly unrelated areas of mathematics.

This connection highlights the unexpected power and depth of VOAs and provides a stunning example of how abstract mathematical structures can reveal profound relationships in seemingly disparate mathematical fields.

Book Outline: "An Introduction to Vertex Operator Algebras"

I. Introduction:
What are Vertex Operator Algebras?
Motivation and Applications
Historical Context

II. Foundations of Vertex Operator Algebras:
Definition and Axioms (Borcherds Identities)
Key Examples (e.g., Heisenberg VOA, Virasoro VOA)
Basic Properties and Structures

III. Representations of Vertex Operator Algebras:
Modules and their Classification
Highest-Weight Representations
Character Theory and Modular Forms

IV. Conformal Field Theory and Vertex Operator Algebras:
Connection to CFT
Vertex Operators and Conformal Transformations
Applications in Physics

V. Monstrous Moonshine and Beyond:
The Monster Group and Modular Invariants
The Moonshine Module VOA
Open Problems and Future Directions

VI. Conclusion:
Summary of Key Concepts
Research Directions and Applications

(Detailed explanation of each point in the outline would constitute a full textbook, so I'll provide a concise expansion of some key points):

II. Foundations of VOAs: This section would provide a rigorous definition of a VOA, explaining the Borcherds identities and proving basic properties. Examples like the Heisenberg VOA (related to free bosons) and the Virasoro VOA (encoding conformal symmetry) would be carefully constructed and analyzed.

III. Representations of VOAs: This chapter would delve into the theory of VOA modules. The notion of highest-weight modules would be explained, along with techniques for classifying these representations. The connection to characters and modular forms would be explored.

IV. Conformal Field Theory and VOAs: This section would formally connect VOAs to CFT, showing how vertex operators in a VOA correspond to operators in a CFT. Examples of specific CFTs constructed from VOAs would be discussed.

V. Monstrous Moonshine: This section would discuss the historical context of Moonshine, presenting the conjecture and outlining its proof using the Moonshine module VOA.

Frequently Asked Questions (FAQs):

1. What is the significance of the Borcherds identities in the definition of a VOA? They are the defining axioms, ensuring the consistency and rich structure of the algebra of vertex operators.

1. How are VOAs related to Lie algebras? VOAs can be seen as a sophisticated generalization of Lie algebras, capturing a more intricate structure.

1. What is the central charge in a VOA? It's a crucial invariant, often related to the Virasoro algebra present within many VOAs, and plays a significant role in physical applications.

1. What are highest-weight representations, and why are they important? These are specific representations built from a "highest-weight vector," crucial for analyzing VOA structure.

1. How do VOAs relate to conformal field theory? VOAs provide a rigorous mathematical framework for understanding the algebraic structure of CFTs.

1. What is the moonshine module? It's a specific VOA that provided the key to understanding the monstrous moonshine conjecture.

1. Are there any applications of VOAs beyond theoretical physics? While heavily used in physics, they are increasingly used in other areas of mathematics like number theory.

1. What are some open problems in the study of VOAs? Classifying all VOAs, understanding their representation theory completely, and finding further applications remain active research areas.

1. Where can I learn more about VOAs? Start with introductory texts on CFT and look for advanced texts focusing on VOAs and related algebraic structures.

Related Articles:

1. Introduction to Conformal Field Theory: Provides the necessary background knowledge in CFT, essential for understanding the application of VOAs.

1. Lie Algebras and their Representations: Explains the fundamental concepts of Lie algebras, paving the way to understanding the more complex structure of VOAs.

1. Modular Forms and their Applications: Covers the theory of modular forms, relevant to the connection between VOAs and monstrous moonshine.

1. The Monster Group: A Mathematical Enigma: Explores the properties of the Monster group, essential for understanding the monstrous moonshine conjecture.

1. String Theory for Beginners: Provides a basic introduction to string theory, where VOAs play a crucial role.

1. The Borcherds Identities: A Deeper Dive: Explores the technical details of the Borcherds identities, the axioms defining a VOA.

1. Highest-Weight Representations of VOAs: Provides a more in-depth look at the theory of highest-weight representations.

1. Applications of VOAs in Physics Beyond String Theory: Examines the potential applications of VOAs in other areas of physics.

1. Recent Advances in Vertex Operator Algebra Theory: Summarizes recent research results and open problems in the field.

vertex operator algebras: Vertex Operator Algebras and the Monster Igor Frenkel, James Lepowsky, Arne Meurman, 1989-05-01 This work is motivated by and develops connections between several branches of mathematics and physics--the theories of Lie algebras, finite groups and modular functions in mathematics, and string theory in physics. The first part of the book presents a new mathematical theory of vertex operator algebras, the algebraic counterpart of two-dimensional holomorphic conformal quantum field theory. The remaining part constructs the Monster finite simple group as the automorphism group of a very special vertex operator algebra, called the moonshine module because of its relevance to monstrous moonshine.

vertex operator algebras: *Introduction to Vertex Operator Algebras and Their Representations* James Lepowsky, Haisheng Li, 2012-12-06 * Introduces the fundamental theory of vertex operator algebras and its basic techniques and examples. * Begins with a detailed presentation of the theoretical foundations and proceeds to a range of applications. * Includes a number of new, original results and brings fresh perspective to important works of many other researchers in algebra, lie theory, representation theory, string theory, quantum field theory, and other areas of math and physics.

vertex operator algebras: *Introduction to Vertex Operator Algebras and Their Representations* James Lepowsky, Haisheng Li, 2004 The deep and relatively new field of vertex operator algebras is intimately related to a variety of areas in mathematics and physics: for example, the concepts of monstrous moonshine, infinite-dimensional Lie theory, string theory, and conformal field theory. This book introduces the reader to the fundamental theory of vertex operator algebras and its basic techniques and examples. Beginning with a detailed presentation of the theoretical foundations and proceeding to a range of applications, the text includes a number of new, original results and also highlights and brings fresh perspective to important works of many researchers. After introducing the elementary formal calculus'' underlying the subject, the book provides an axiomatic development of vertex operator algebras and their modules, expanding on the early contributions of R. Borcherds, I. Frenkel, J. Lepowsky, A. Meurman, Y.-Z. Huang, C. Dong, Y. Zhu and others. The concept of a representation'' of a vertex (operator) algebra is treated in detail, following and extending the work of H. Li; this approach is used to construct important families of vertex (operator) algebras and their modules. Requiring only a familiarity with basic algebra, Introduction to Vertex Operator Algebras and Their Representations will be useful for graduate students and researchers in mathematics and physics. The booka??s presentation of the core topics will equip readers to embark on many active research directions related to vertex operator algebras, group theory, representation theory, and string theory.

vertex operator algebras: Vertex Algebras and Algebraic Curves Edward Frenkel, David Ben-Zvi, 2004-08-25 Vertex algebras are algebraic objects that encapsulate the concept of operator product expansion from two-dimensional conformal field theory. Vertex algebras are fast becoming ubiquitous in many areas of modern mathematics, with applications to representation theory, algebraic geometry, the theory of finite groups, modular functions, topology, integrable systems, and combinatorics. This book is an introduction to the theory of vertex algebras with a particular emphasis on the relationship with the geometry of algebraic curves. The notion of a vertex algebra is introduced in a coordinate-independent way, so that vertex operators become well defined on arbitrary smooth algebraic curves, possibly equipped with additional data, such as a vector bundle. Vertex algebras then appear as the algebraic objects encoding the geometric structure of various moduli spaces associated with algebraic curves. Therefore they may be used to give a geometric interpretation of various questions of representation theory. The book contains many original results, introduces important new concepts, and brings new insights into the theory of vertex algebras. The authors have made a great effort to make the book self-contained and accessible to readers of all backgrounds. Reviewers of the first edition anticipated that it would have a long-lasting influence on this exciting field of mathematics and would be very useful for graduate students and researchers interested in the subject. This second edition, substantially improved and expanded, includes several new topics, in particular an introduction to the Beilinson-Drinfeld theory of factorization algebras and the geometric Langlands correspondence.

vertex operator algebras: Generalized Vertex Algebras and Relative Vertex Operators Chongying Dong, James Lepowsky, 2012-12-06 The rapidly-evolving theory of vertex operator algebras provides deep insight into many important algebraic structures. Vertex operator algebras can be viewed as complex analogues of both Lie algebras and associative algebras. The monograph is written in a n accessible and self-contained manner, with detailed proofs and with many examples interwoven through the axiomatic treatment as motivation and applications. It will be useful for research mathematicians and theoretical physicists working the such fields as representation theory and algebraic structure sand will provide the basis for a number of graduate courses and seminars on these and related topics.

vertex operator algebras: *Vertex Operator Algebras in Mathematics and Physics* Stephen Berman, 2003 Vertex operator algebras are a class of algebras underlying a number of recent constructions, results, and themes in mathematics. These algebras can be understood as ''string-theoretic analogues'' of Lie algebras and of commutative associative algebras. They play fundamental roles in some of the most active research areas in mathematics and physics. Much recent progress in both physics and mathematics has benefited from cross-pollination between the physical and mathematical points of view. This book presents the proceedings from the workshop, ''Vertex Operator Algebras in Mathematics and Physics'', held at The Fields Institute. It consists of papers based on many of the talks given at the conference by leading experts in the algebraic, geometric, and physical aspects of vertex operator algebra theory. The book is suitable for graduate students and research mathematicians interested in the major themes and important developments on the frontier of research in vertex operator algebra theory and its applications in mathematics and physics.

vertex operator algebras: Two-Dimensional Conformal Geometry and Vertex Operator Algebras Yi-Zhi Huang, 2012-12-06 The theory of vertex operator algebras and their representations has been showing its power in the solution of concrete mathematical problems and in the understanding of conceptual but subtle mathematical and physical struc- tures of conformal field theories. Much of the recent progress has deep connec- tions with complex analysis and conformal geometry. Future developments, especially constructions and studies of higher-genus theories, will need a solid geometric theory of vertex operator algebras. Back in 1986, Manin already observed in Man) that the quantum theory of (super)strings existed (in some sense) in two entirely different mathematical fields. Under canonical quantization this theory appeared to a mathematician as the representation theories of the Heisenberg, Vir as oro and affine Kac- Moody algebras and their superextensions. Quantization with the help of the Polyakov path integral led on the other hand to the analytic theory of algebraic (super) curves and their moduli spaces, to invariants of the type of the analytic curvature, and so on.He pointed out further that establishing direct mathematical connections between these two forms of a single theory was a big and important problem. On the one hand, the theory of vertex operator algebras and their repre- sentations unifies (and considerably extends) the representation theories of the Heisenberg, Virasoro and Kac-Moody algebras and their superextensions.

vertex operator algebras: *On Axiomatic Approaches to Vertex Operator Algebras and Modules* Igor Frenkel, Yi-Zhi Huang, James Lepowsky, 1993 The basic definitions and properties of vertex operator algebras, modules, intertwining operators and related concepts are presented, following a fundamental analogy with Lie algebra theory. The first steps in the development of the general theory are taken, and various natural and useful reformulations of the axioms are given. In particular, tensor products of algebras and modules, adjoint vertex operators and contragradient modules, adjoint intertwining operators and fusion rules are studied in greater depth. This paper lays the monodromy-free axiomatic foundation of the general theory of vertex operator algebras, modules and intertwining operators.

vertex operator algebras: *Spinor Construction of Vertex Operator Algebras, Triality, and $E^{(1)}_8$* Alex J. Feingold, Igor Frenkel, John F. X. Ries, 1991 The theory of vertex operator algebras is a remarkably rich new mathematical field which captures the algebraic content of conformal field theory in physics. Ideas leading up to this theory appeared in physics as part of statistical mechanics and string theory. In mathematics, the axiomatic definitions crystallized in the work of Borcherds and in Vertex Operator Algebras and the Monster, by Frenkel, Lepowsky, and Meurman. The structure of monodromies of intertwining operators for modules of vertex operator algebras yield braid group representations and leads to natural generalizations of vertex operator algebras, such as superalgebras and para-algebras. Many examples of vertex operator algebras and their generalizations are related to constructions in classical representation theory and shed new light on the classical theory. This book accomplishes several goals. The authors provide an explicit spinor construction, using only Clifford algebras, of a vertex operator superalgebra structure on the direct sum of the basic and vector modules for the affine Kac-Moody algebra Dn(1). They also review and extend Chevalley's spinor construction of the 24-dimensional commutative nonassociative algebraic structure and triality on the direct sum of the three 8-dimensional D4-modules. Vertex operator para-algebras, introduced and developed independently in this book and by Dong and Lepowsky, are related to one-dimensional representations of the braid group. The authors also provide a unified approach to the Chevalley, Greiss, and E8 algebras and explain some of their similarities. A Third goal is to provide a purely spinor construction of the exceptional affine Lie algebra E8(1), a natural continuation of previous work on spinor and oscillator constructions of the classical affine Lie algebras. These constructions should easily extend to include the rest of the exceptional affine Lie algebras. The final objective is to develop an inductive technique of construction which could be applied to the Monster vertex operator algebra. Directed at mathematicians and physicists, this book should be accessible to graduate students with some background in finite-dimensional Lie algebras and their representations. Although some experience with affine Kac-Moody algebras would be useful, a summary of the relevant parts of that theory is included. This book shows how the concepts and techniques of Lie theory can be generalized to yield the algebraic structures associated with conformal field theory. The careful reader will also gain a detailed knowledge of how the spinor construction of classical triality lifts to the affine algebras and plays an important role in the spinor construction of vertex operator algebras, modules, and intertwining operators with nontrivial monodromies.

vertex operator algebras: Vertex Algebras for Beginners Victor G. Kac, 1998 Based on courses given by the author at MIT and at Rome University in spring 1997, this book presents an introduction to algebraic aspects of conformal field theory. It includes material on the foundations of a rapidly growing area of algebraic conformal theory.

vertex operator algebras: Lie Algebras, Vertex Operator Algebras and Their Applications Yi-Zhi Huang, Kailash C. Misra, 2007 The articles in this book are based on talks given at the international conference 'Lie algebras, vertex operator algebras and their applications'. The focus of the papers is mainly on Lie algebras, quantum groups, vertex operator algebras and their applications to number theory, combinatorics and conformal field theory.

vertex operator algebras: Vertex Operators in Mathematics and Physics J. Lepowsky, S.

Mandelstam, I.M. Singer, 2013-03-08 James Lepowsky t The search for symmetry in nature has for a long time provided representation theory with perhaps its chief motivation. According to the standard approach of Lie theory, one looks for infinitesimal symmetry -- Lie algebras of operators or concrete realizations of abstract Lie algebras. A central theme in this volume is the construction of affine Lie algebras using formal differential operators called vertex operators, which originally appeared in the dual-string theory. Since the precise description of vertex operators, in both mathematical and physical settings, requires a fair amount of notation, we do not attempt it in this introduction. Instead we refer the reader to the papers of Mandelstam, Goddard-Olive, Lepowsky-Wilson and Frenkel-Lepowsky-Meurman. We have tried to maintain consistency of terminology and to some extent notation in the articles herein. To help the reader we shall review some of the terminology. We also thought it might be useful to supplement an earlier fairly detailed exposition of ours [37] with a brief historical account of vertex operators in mathematics and their connection with affine algebras. Since we were involved in the development of the subject, the reader should be advised that what follows reflects our own understanding. For another view, see [29].1 t Partially supported by the National Science Foundation through the Mathematical Sciences Research Institute and NSF Grant MCS 83-01664. 1 We would like to thank Igor Frenkel for his valuable comments on the first draft of this introduction.

vertex operator algebras: String Path Integral Realization of Vertex Operator Algebras Haruo Tsukada, 1991 We establish relations between vertex operator algebras in mathematics and string path integrals in physics. In particular, we construct the basic representations of affine Lie algebras of [italic capitals]ÂĎÊ-type using a method of string path integrals.

vertex operator algebras: Lie Algebras, Vertex Operator Algebras, and Related Topics Katrina Barron, Elizabeth Jurisich, Antun Milas, Kailash Misr, 2017-08-15 This volume contains the proceedings of the conference on Lie Algebras, Vertex Operator Algebras, and Related Topics, celebrating the 70th birthday of James Lepowsky and Robert Wilson, held from August 14–18, 2015, at the University of Notre Dame, Notre Dame, Indiana. Since their seminal work in the 1970s, Lepowsky and Wilson, their collaborators, their students, and those inspired by their work, have developed an amazing body of work intertwining the fields of Lie algebras, vertex algebras, number theory, theoretical physics, quantum groups, the representation theory of finite simple groups, and more. The papers presented here include recent results and descriptions of ongoing research initiatives representing the broad influence and deep connections brought about by the work of Lepowsky and Wilson and include a contribution by Yi-Zhi Huang summarizing some major open problems in these areas, in particular as they pertain to two-dimensional conformal field theory.

vertex operator algebras: From Vertex Operator Algebras to Conformal Nets and Back Sebastiano Carpi, Yasuyuki Kawahigashi, Roberto Longo, Mihály Weiner, 2018-08-09 The authors consider unitary simple vertex operator algebras whose vertex operators satisfy certain energy bounds and a strong form of locality and call them strongly local. They present a general procedure which associates to every strongly local vertex operator algebra V a conformal net AV acting on the Hilbert space completion of V and prove that the isomorphism class of AV does not depend on the choice of the scalar product on V. They show that the class of strongly local vertex operator algebras is closed under taking tensor products and unitary subalgebras and that, for every strongly local vertex operator algebra V, the map W↦AW gives a one-to-one correspondence between the unitary subalgebras W of V and the covariant subnets of AV.

vertex operator algebras: Vertex operator algebras in mathematics and physics Stephen Berman, 2003 This book presents the proceedings from the workshop, Vertex Operator Algebras in Mathematics and Physics, held at The Fields Institute. It consists of papers based on many of the talks given at the conference by leading experts in the algebraic, geometric, and physical aspects of vertex operator algebra theory. The book is suitable for graduate students and research mathematicians interested in the major themes and important developments on the frontier of research in vertex operator algebra theory and its applications in mathematics and physics.

vertex operator algebras: *A Mathematical Introduction to Conformal Field Theory* Martin

Schottenloher, 2008-09-15 Part I gives a detailed, self-contained and mathematically rigorous exposition of classical conformal symmetry in n dimensions and its quantization in two dimensions. The conformal groups are determined and the appearence of the Virasoro algebra in the context of the quantization of two-dimensional conformal symmetry is explained via the classification of central extensions of Lie algebras and groups. Part II surveys more advanced topics of conformal field theory such as the representation theory of the Virasoro algebra, conformal symmetry within string theory, an axiomatic approach to Euclidean conformally covariant quantum field theory and a mathematical interpretation of the Verlinde formula in the context of moduli spaces of holomorphic vector bundles on a Riemann surface.

vertex operator algebras: Moonshine beyond the Monster Terry Gannon, 2023-07-31

vertex operator algebras: *Proceedings Of The International Congress Of Mathematicians 2018 (Icm 2018) (In 4 Volumes)* Boyan Sirakov, Paulo Ney De Souza, Marcelo Viana, 2019-02-27 The Proceedings of the ICM publishes the talks, by invited speakers, at the conference organized by the International Mathematical Union every 4 years. It covers several areas of Mathematics and it includes the Fields Medal and Nevanlinna, Gauss and Leelavati Prizes and the Chern Medal laudatios.

vertex operator algebras: *Elliptic Genera and Vertex Operator Super-Algebras* Hirotaka Tamanoi, 2006-11-14 This monograph deals with two aspects of the theory of elliptic genus: its topological aspect involving elliptic functions, and its representation theoretic aspect involving vertex operator super-algebras. For the second aspect, elliptic genera are shown to have the structure of modules over certain vertex operator super-algebras. The vertex operators corresponding to parallel tensor fields on closed Riemannian Spin Kähler manifolds such as Riemannian tensors and Kähler forms are shown to give rise to Virasoro algebras and affine Lie algebras. This monograph is chiefly intended for topologists and it includes accounts on topics outside of topology such as vertex operator algebras.

vertex operator algebras: *Vertex Operator Algebras, Number Theory and Related Topics* Matthew Krauel, Michael Tuite, Gaywalee Yamskulna, 2020-07-13 This volume contains the proceedings of the International Conference on Vertex Operator Algebras, Number Theory, and Related Topics, held from June 11–15, 2018, at California State University, Sacramento, California. The mathematics of vertex operator algebras, vector-valued modular forms and finite group theory continues to provide a rich and vibrant landscape in mathematics and physics. The resurgence of moonshine related to the Mathieu group and other groups, the increasing role of algebraic geometry and the development of irrational vertex operator algebras are just a few of the exciting and active areas at present. The proceedings center around active research on vertex operator algebras and vector-valued modular forms and offer original contributions to the areas of vertex algebras and number theory, surveys on some of the most important topics relevant to these fields, introductions to new fields related to these and open problems from some of the leaders in these areas.

vertex operator algebras: Tensor Categories for Vertex Operator Superalgebra Extensions Thomas Creutzig, Shashank Kanade, Robert McRae, 2024-04-17 View the abstract.

vertex operator algebras: *Rational Vertex Operator Algebras with Small Central Charges and Generators* Tangjiasanshao, 2008

vertex operator algebras: Operator Algebras and Mathematical Physics Masaki Izumi, Yasuyuki Kawahigashi, Motoko Kotani, Hiroki Matui, Narutaka Ozawa, 2019 This is a proceedings volume of an international conference 'Operator Algebras and Mathematical Physics' held at Tohoku University in August, 2016. This meeting was the 9th MSJ-Seasonal Institute of the Mathematical Society of Japan. Twenty-eight researchers gave lectures on a wide range of topics on operator algebras and their applications to mathematical physics. This volume contains one survey article and 11 research articles based on the lectures given. We recommend this to researchers and graduate students in related fields strongly.Published by Mathematical Society of Japan and distributed by World Scientific Publishing Co. for all markets except North America

vertex operator algebras: Introduction to Vertex Operator Superalgebras and Their Modules

Xiaoping Xu, 2013-03-09 This book presents a systematic study on the structures of vertex operator superalgebras and their modules. Related theories of self-dual codes and lattices are included, as well as recent achievements on classifications of certain simple vertex operator superalgebras and their irreducible twisted modules, constructions of simple vertex operator superalgebras from graded associative algebras and their anti-involutions, self-dual codes and lattices. Audience: This book is of interest to researchers and graduate students in mathematics and mathematical physics.

vertex operator algebras: Tensor Categories Pavel Etingof, Shlomo Gelaki, Dmitri Nikshych, Victor Ostrik, 2016-08-05 Is there a vector space whose dimension is the golden ratio? Of course not—the golden ratio is not an integer! But this can happen for generalizations of vector spaces—objects of a tensor category. The theory of tensor categories is a relatively new field of mathematics that generalizes the theory of group representations. It has deep connections with many other fields, including representation theory, Hopf algebras, operator algebras, low-dimensional topology (in particular, knot theory), homotopy theory, quantum mechanics and field theory, quantum computation, theory of motives, etc. This book gives a systematic introduction to this theory and a review of its applications. While giving a detailed overview of general tensor categories, it focuses especially on the theory of finite tensor categories and fusion categories (in particular, braided and modular ones), and discusses the main results about them with proofs. In particular, it shows how the main properties of finite-dimensional Hopf algebras may be derived from the theory of tensor categories. Many important results are presented as a sequence of exercises, which makes the book valuable for students and suitable for graduate courses. Many applications, connections to other areas, additional results, and references are discussed at the end of each chapter.

vertex operator algebras: Topological Field Theory, Primitive Forms and Related Topics A. Kashiwara, A. Matsuo, K. Saito, I. Satake, 2012-12-06 As the interaction of mathematics and theoretical physics continues to intensify, the theories developed in mathematics are being applied to physics, and conversely. This book centers around the theory of primitive forms which currently plays an active and key role in topological field theory (theoretical physics), but was originally developed as a mathematical notion to define a good period mapping for a family of analytic structures. The invited papers in this volume are expository in nature by participants of the Taniguchi Symposium on Topological Field Theory, Primitive Forms and Related Topics and the RIMS Symposium bearing the same title, both held in Kyoto. The papers reflect the broad research of some of the world's leading mathematical physicists, and should serve as an excellent resource for researchers as well as graduate students of both disciplines.

vertex operator algebras: Kac-Moody and Virasoro Algebras Peter Goddard, David Olive, 1988 This volume reviews the subject of Kac-Moody and Virasoro Algebras. It serves as a reference book for physicists with commentary notes and reprints.

vertex operator algebras: Crossed Products of Operator Algebras Elias G. Katsoulis, Christopher Ramsey, 2019-04-10 The authors study crossed products of arbitrary operator algebras by locally compact groups of completely isometric automorphisms. They develop an abstract theory that allows for generalizations of many of the fundamental results from the selfadjoint theory to our context. They complement their generic results with the detailed study of many important special cases. In particular they study crossed products of tensor algebras, triangular AF algebras and various associated C -algebras. They make contributions to the study of C -envelopes, semisimplicity, the semi-Dirichlet property, Takai duality and the Hao-Ng isomorphism problem. They also answer questions from the pertinent literature.

vertex operator algebras: Lie Groups, Geometry, and Representation Theory Victor G. Kac, Vladimir L. Popov, 2018-12-12 This volume, dedicated to the memory of the great American mathematician Bertram Kostant (May 24, 1928 – February 2, 2017), is a collection of 19 invited papers by leading mathematicians working in Lie theory, representation theory, algebra, geometry, and mathematical physics. Kostant's fundamental work in all of these areas has provided deep new insights and connections, and has created new fields of research. This volume features the only published articles of important recent results of the contributors with full details of their proofs. Key topics include: Poisson structures and potentials (A. Alekseev, A. Berenstein, B. Hoffman) Vertex algebras (T. Arakawa, K. Kawasetsu) Modular irreducible representations of semisimple Lie algebras (R. Bezrukavnikov, I. Losev) Asymptotic Hecke algebras (A. Braverman, D. Kazhdan) Tensor categories and quantum groups (A. Davydov, P. Etingof, D. Nikshych) Nil-Hecke algebras and Whittaker D-modules (V. Ginzburg) Toeplitz operators (V. Guillemin, A. Uribe, Z. Wang) Kashiwara crystals (A. Joseph) Characters of highest weight modules (V. Kac, M. Wakimoto) Alcove polytopes (T. Lam, A. Postnikov) Representation theory of quantized Gieseker varieties (I. Losev) Generalized Bruhat cells and integrable systems (J.-H. Liu, Y. Mi) Almost characters (G. Lusztig) Verlinde formulas (E. Meinrenken) Dirac operator and equivariant index (P.-É. Paradan, M. Vergne) Modality of representations and geometry of θ-groups (V. L. Popov) Distributions on homogeneous spaces (N. Ressayre) Reduction of orthogonal representations (J.-P. Serre)

vertex operator algebras: Infinite Dimensional Lie Algebras Victor G. Kac, 2013-11-09

vertex operator algebras: Langlands Correspondence for Loop Groups Edward Frenkel, 2007-06-28 The first account of local geometric Langlands Correspondence, a new area of mathematical physics developed by the author.

vertex operator algebras: *Factorization Algebras in Quantum Field Theory* Kevin Costello, Owen Gwilliam, 2017 This first volume develops factorization algebras with a focus upon examples exhibiting their use in field theory, which will be useful for researchers and graduates.

vertex operator algebras: *Recent Developments in Quantum Affine Algebras and Related Topics* Naihuan Jing, Kailash C. Misra, 1999 This volume reflects the proceedings of the International Conference on Representations of Affine and Quantum Affine Algebras and Their Applications held at North Carolina State University (Raleigh). In recent years, the theory of affine and quantum affine Lie algebras has become an important area of mathematical research with numerous applications in other areas of mathematics and physics. Three areas of recent progress are the focus of this volume: affine and quantum affine algebras and their generalizations, vertex operator algebras and their representations, and applications in combinatorics and statistical mechanics. Talks given by leading international experts at the conference offered both overviews on the subjects and current research results. The book nicely presents the interplay of these topics recently occupying centre stage in the theory of infinite dimensional Lie theory.

vertex operator algebras: Vertex Operator Algebras and Dual Pairs Gaywalee Yamskulna, 2001

vertex operator algebras: Affine Lie Algebras and Quantum Groups Jürgen Fuchs, 1995-03-09 This is an introduction to the theory of affine Lie Algebras, to the theory of quantum groups, and to the interrelationships between these two fields that are encountered in conformal field theory.

vertex operator algebras: On Axiomatic Approaches to Vertex Operator Algebras and Modules Igor Frenkel, Yi-Zhi Huang, James Lepowsky, The notion of vertex operator algebra arises naturally in the vertex operator construction of the Monster---the largest sporadic finite simple group. From another perspective, the theory of vertex operator algebras and their modules forms the algebraic foundation of conformal field theory. Vertex operator algebras and conformal field theory are now known to be deeply related to many important areas of mathematics. This essentially self-contained monograph develops the basic axiomatic theory of vertex operator algebras and their modules and intertwining operators, following a fundamental analogy with Lie algebra theory. The main axiom, the ''Jacobi(-Cauchy) identity'', is a far-reaching analog of the Jacobi identity for Lie algebras. The authors show that the Jacobi identity is equivalent to suitably formulated rationality, commutativity, and associativity properties of products of quantum fields. A number of other foundational and useful results are also developed. This work was originally distributed as a preprint in 1989, and in view of the current widespread interest in the subject among mathematicians and theoretical physicists, its publication and availability should prove no less useful than when it was written.

vertex operator algebras: Lie Theory and Geometry Jean-Luc Brylinski, Ranee Brylinski, Victor

Guillemin, Victor Kac, 2012-12-06 This volume, dedicated to Bertram Kostant on the occasion of his 65th birthday, is a collection of 22 invited papers by leading mathematicians working in Lie theory, geometry, algebra, and mathematical physics. Kostant's fundamental work in all these areas has provided deep new insights and connections, and has created new fields of research. The papers gathered here present original research articles as well as expository papers, broadly reflecting the range of Kostant's work.

vertex operator algebras: Noncommutative Geometry Alain Connes, Joachim Cuntz, Erik G. Guentner, Nigel Higson, Jerome Kaminker, John E. Roberts, 2003-12-15 Noncommutative Geometry is one of the most deep and vital research subjects of present-day Mathematics. Its development, mainly due to Alain Connes, is providing an increasing number of applications and deeper insights for instance in Foliations, K-Theory, Index Theory, Number Theory but also in Quantum Physics of elementary particles. The purpose of the Summer School in Martina Franca was to offer a fresh invitation to the subject and closely related topics; the contributions in this volume include the four main lectures, cover advanced developments and are delivered by prominent specialists.

vertex operator algebras: Kac-Moody Groups, their Flag Varieties and Representation Theory Shrawan Kumar, 2002-09-10 Most of these topics appear here for the first time in book form. Many of them are interesting even in the classical case of semi-simple algebraic groups. Some appendices recall useful results from other areas, so the work may be considered self-contained, although some familiarity with semi-simple Lie algebras or algebraic groups is helpful. It is clear that this book is a valuable reference for all those interested in flag varieties and representation theory in the semi-simple or Kac-Moody case. —MATHEMATICAL REVIEWS A lot of different topics are treated in this monumental work. . . . many of the topics of the book will be useful for those only interested in the finite-dimensional case. The book is self contained, but is on the level of advanced graduate students. . . . For the motivated reader who is willing to spend considerable time on the material, the book can be a gold mine. —ZENTRALBLATT MATH

Additional Resources

vertex operator algebras

Vertex Operator Algebras

Vertex Operator Algebras

Related Articles

- weekly language review q2 2 answer key
- unit 2 lesson 7 cumulative practice problems answer key
- virago wellness initiative appleton wi

Back to Home