

rockafellar convex analysis

Rockafellar Convex Analysis: A Deep Dive into the Fundamentals

Introduction:

Are you fascinated by the elegance and power of optimization problems? Do you yearn to understand the theoretical underpinnings of algorithms that drive modern machine learning and data science? Then you've come to the right place. This comprehensive guide delves into the world of Rockafellar's Convex Analysis, a seminal work that laid the foundation for much of modern convex optimization theory. We'll unpack its core concepts, explore key theorems, and demonstrate its practical applications. This post isn't just a summary; it's a journey into the heart of a mathematical masterpiece, equipping you with a solid understanding of this crucial field. Prepare to unlock the secrets of convexity!

What is Convex Analysis?

Before diving into Rockafellar's contribution, let's establish a foundational understanding. Convex analysis is a branch of mathematics that studies convex sets and convex functions. A set is convex if a line segment connecting any two points within the set remains entirely within the set. Similarly, a function is convex if the line segment connecting any two points on its graph lies above the graph itself. These seemingly simple definitions have profound consequences, leading to powerful properties and algorithms.

Convexity is crucial because it guarantees the existence and uniqueness of solutions in many optimization problems. Unlike non-convex problems, which can have multiple local optima, convex problems possess a single global optimum, making them significantly easier to solve. This characteristic has made convex analysis indispensable in various fields, including:

Machine Learning: Many machine learning algorithms rely on convex optimization for training models, ensuring efficient and accurate results.

Operations Research: Solving problems related to resource allocation, scheduling, and logistics often involves convex optimization techniques.

Economics: Convex analysis plays a crucial role in modeling economic systems and optimizing resource allocation.

Engineering: Design optimization problems in various engineering disciplines frequently leverage the properties of convexity.

Rockafellar's Contribution: A Landmark Text

R. Tyrrell Rockafellar's "Convex Analysis," published in 1970, is considered the definitive text on the subject. It's not an easy read—it demands a strong mathematical background—but it's a rewarding one for those willing to invest the effort. Rockafellar systematically developed the theory of convex analysis, introducing and rigorously proving fundamental theorems and concepts that remain central to the field today. His work unified and extended previous results, creating a cohesive and powerful framework for understanding and solving convex optimization problems.

Key Concepts in Rockafellar's Convex Analysis:

Convex Sets and Their Properties: Rockafellar meticulously explores the properties of convex sets, including their closures, interiors, and relative interiors. He establishes fundamental results concerning the separation of convex sets and the representation of convex sets as intersections of half-spaces.

Convex Functions and Their Properties: The book dives deep into the properties of convex functions, including subgradients, conjugate functions, and their relationships. The concept of the subgradient, a generalization of the gradient for non-differentiable functions, is central to many optimization algorithms.

Conjugate Functions and Duality: The concept of conjugate functions plays a pivotal role in Rockafellar's framework. Conjugate duality is a powerful tool for analyzing and solving optimization problems, providing alternative perspectives and enabling the development of efficient algorithms.

Subdifferentials and Optimization: The subdifferential, the set of all subgradients at a point, is extensively explored. Rockafellar demonstrates its importance in characterizing optimality conditions and developing iterative algorithms for solving convex optimization problems.

Applications and Extensions: While rigorous in its mathematical treatment, Rockafellar's work doesn't shy away from showcasing the practical applications of convex analysis. He demonstrates how the developed theory can be applied to various optimization problems, providing a bridge between theory and practice.

A Structured Outline of Rockafellar's Convex Analysis:

Book Title: Convex Analysis

Outline:

Introduction: Overview of convexity, its importance, and the book's scope.

Chapter 1: Convex Sets: Definitions, properties, separation theorems, and topological aspects of convex sets.

Chapter 2: Convex Functions: Definitions, properties, subgradients, directional derivatives, and continuity.

Chapter 3: Conjugate Functions: Definition, properties, Fenchel duality, and applications to optimization.

Chapter 4: Subdifferentials: Detailed analysis of subdifferentials, their properties, and relationships to optimality conditions.

Chapter 5: Applications to Optimization: Illustrative examples and applications of convex analysis to various optimization problems.

Chapter 6: Extensions and Advanced Topics: More advanced concepts and extensions of the theory (e.g., generalized convexity).

Conclusion: Summary of key concepts and their significance in the field of optimization.

Appendix: Mathematical preliminaries and notations.

Detailed Explanation of the Outline:

(1) Introduction: This section lays the groundwork, providing a clear definition of convexity and highlighting its importance in optimization. It sets the stage for the more technical chapters to follow.

(2) Chapter 1: Convex Sets: This chapter delves into the fundamental building blocks—convex sets. It

rigorously defines convexity and explores various properties, including convex hulls, separation theorems (crucial for characterizing solutions), and topological considerations related to closure and interior points. Understanding this chapter is crucial for grasping subsequent material.

(3) Chapter 2: Convex Functions: Here, the focus shifts to convex functions. This chapter defines convex functions, explores their properties (e.g., continuity, differentiability), introduces the crucial concept of subgradients (generalizing gradients for non-differentiable functions), and examines directional derivatives. Mastering this chapter is essential for understanding optimization algorithms based on subgradients.

(4) Chapter 3: Conjugate Functions: This chapter introduces the powerful concept of conjugate functions, a pivotal tool in duality theory. It explains how to compute conjugate functions, demonstrates their properties, and showcases their role in Fenchel duality, a cornerstone of convex optimization. Understanding conjugate functions is essential for developing advanced optimization algorithms.

(5) Chapter 4: Subdifferentials: This chapter provides an in-depth analysis of subdifferentials—the sets of all subgradients at a given point. Rockafellar meticulously explores their properties, establishes their connections to optimality conditions (necessary and sufficient conditions for a point to be a minimizer), and lays the foundation for many optimization algorithms.

(6) Chapter 5: Applications to Optimization: This chapter bridges the gap between theory and practice. It provides concrete examples of how the theoretical tools developed in the preceding chapters can be applied to solve various optimization problems. It serves as a practical demonstration of the power of convex analysis.

(7) Chapter 6: Extensions and Advanced Topics: This chapter delves into more specialized and advanced topics, exploring extensions of the core concepts and addressing more complex scenarios. This could include generalized notions of convexity or applications to specific types of optimization problems.

(8) Conclusion: This section summarizes the key contributions and highlights the lasting impact of Rockafellar's work on the field of convex optimization.

(9) Appendix: This section provides a comprehensive list of mathematical preliminaries and notations used throughout the book, ensuring clarity and accessibility for the reader.

FAQs:

1. What is the prerequisite knowledge needed to understand Rockafellar's Convex Analysis? A strong background in linear algebra, real analysis, and some familiarity with topology is essential.
2. Is Rockafellar's book suitable for beginners in convex optimization? No, it's a highly advanced text best suited for those with a solid mathematical foundation.
3. What are the main applications of Rockafellar's Convex Analysis? Machine learning, operations research, economics, and engineering.
4. What is the significance of conjugate functions in Rockafellar's work? They are crucial for duality

theory and developing efficient optimization algorithms.

5. What are subgradients and why are they important? They are generalizations of gradients for non-differentiable functions and are essential for characterizing optimality conditions.
6. How does Rockafellar's book contribute to the field of optimization? It provides a rigorous and unified framework for understanding and solving convex optimization problems.
7. Are there any online resources that can help in understanding Rockafellar's Convex Analysis? While the book itself is the primary resource, online lecture notes and supplementary materials from universities can be helpful.
8. What are some alternative books on convex analysis that are more accessible to beginners? Several introductory texts offer a gentler approach to the subject.
9. Is it necessary to read the entire book to understand the core concepts? While reading the entire book is ideal, focusing on key chapters and concepts can provide a solid understanding of the fundamentals.

Related Articles:

1. Introduction to Convex Optimization: A beginner-friendly overview of the field.
2. Subgradient Methods in Machine Learning: Explores algorithms using subgradients.
3. Fenchel Duality and its Applications: A deeper dive into the power of conjugate duality.
4. Convex Optimization in Machine Learning Algorithms: Focuses on practical applications in ML.
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6. Geometric Interpretation of Convex Sets: A visual approach to understanding convex sets.
7. Applications of Convex Analysis in Economics: Explores its role in economic modeling.
8. Solving Convex Optimization Problems with Python: A practical tutorial on implementing algorithms.
9. Advanced Topics in Convex Analysis: Delves into more complex concepts and extensions.

This comprehensive guide provides a solid foundation for understanding Rockafellar's influential work. While the book itself is a challenging read, this article serves as a valuable starting point for those seeking to delve into the fascinating world of convex analysis and its powerful applications.

Remember to consult the original text for a complete and rigorous understanding.

rockafellar convex analysis: Convex Analysis R. Tyrrell Rockafellar, 1997-01-12 Topics treat systems of inequalities; Lagrange multipliers; minimax theorems and duality; structures of convex sets and functions; and more. Available for the first time in paperback, Rockafellar's classic study has firmly established a vital area not only for pure mathematics but also for applications to economics and engineering. Readers will find sound knowledge of linear algebra and introductory real analysis a major benefit to the assimilation of this work.

rockafellar convex analysis: Convex Analysis R. Tyrrell Rockafellar, 1997

rockafellar convex analysis: Convex Analysis R. Tyrrell Rockafellar, 1972

rockafellar convex analysis: Convex Analysis and Optimization Dimitri Bertsekas, Angelia Nedic, Asuman Ozdaglar, 2003-03-01 A uniquely pedagogical, insightful, and rigorous treatment of the analytical/geometrical foundations of optimization. The book provides a comprehensive development of convexity theory, and its rich applications in optimization, including duality, minimax/saddle point theory, Lagrange multipliers, and Lagrangian relaxation/nondifferentiable optimization. It is an excellent supplement to several of our books: Convex Optimization Theory (Athena Scientific, 2009), Convex Optimization Algorithms (Athena Scientific, 2015), Nonlinear Programming (Athena Scientific, 2016), Network Optimization (Athena Scientific, 1998), and Introduction to Linear Optimization (Athena Scientific, 1997). Aside from a thorough account of convex analysis and optimization, the book aims to restructure the theory of the subject, by introducing several novel unifying lines of analysis, including: 1) A unified development of minimax theory and constrained optimization duality as special cases of duality between two simple geometrical problems. 2) A unified development of conditions for existence of solutions of convex optimization problems, conditions for the minimax equality to hold, and conditions for the absence of a duality gap in constrained optimization. 3) A unification of the major constraint qualifications allowing the use of Lagrange multipliers for nonconvex constrained optimization, using the notion of constraint pseudonormality and an enhanced form of the Fritz John necessary optimality conditions. Among its features the book: a) Develops rigorously and comprehensively the theory of convex sets and functions, in the classical tradition of Fenchel and Rockafellar b) Provides a geometric, highly visual treatment of convex and nonconvex optimization problems, including existence of solutions, optimality conditions, Lagrange multipliers, and duality c) Includes an insightful and comprehensive presentation of minimax theory and zero sum games, and its connection with duality d) Describes dual optimization, the associated computational methods, including the novel incremental subgradient methods, and applications in linear, quadratic, and integer programming e) Contains many examples, illustrations, and exercises with complete solutions (about 200 pages) posted at the publisher's web site <http://www.athenasc.com/convexity.html>

rockafellar convex analysis: Convex Analysis and Monotone Operator Theory in Hilbert Spaces Heinz H. Bauschke, Patrick L. Combettes, 2017-02-28 This reference text, now in its second edition, offers a modern unifying presentation of three basic areas of nonlinear analysis: convex analysis, monotone operator theory, and the fixed point theory of nonexpansive operators. Taking a unique comprehensive approach, the theory is developed from the ground up, with the rich connections and interactions between the areas as the central focus, and it is illustrated by a large number of examples. The Hilbert space setting of the material offers a wide range of applications while avoiding the technical difficulties of general Banach spaces. The authors have also drawn upon recent advances and modern tools to simplify the proofs of key results making the book more accessible to a broader range of scholars and users. Combining a strong emphasis on applications with exceptionally lucid writing and an abundance of exercises, this text is of great value to a large audience including pure and applied mathematicians as well as researchers in engineering, data science, machine learning, physics, decision sciences, economics, and inverse problems. The second edition of Convex Analysis and Monotone Operator Theory in Hilbert Spaces greatly expands on the

first edition, containing over 140 pages of new material, over 270 new results, and more than 100 new exercises. It features a new chapter on proximity operators including two sections on proximity operators of matrix functions, in addition to several new sections distributed throughout the original chapters. Many existing results have been improved, and the list of references has been updated. Heinz H. Bauschke is a Full Professor of Mathematics at the Kelowna campus of the University of British Columbia, Canada. Patrick L. Combettes, IEEE Fellow, was on the faculty of the City University of New York and of Université Pierre et Marie Curie – Paris 6 before joining North Carolina State University as a Distinguished Professor of Mathematics in 2016.

rockafellar convex analysis: *Convex Optimization* Stephen P. Boyd, Lieven Vandenberghe, 2004-03-08 Convex optimization problems arise frequently in many different fields. This book provides a comprehensive introduction to the subject, and shows in detail how such problems can be solved numerically with great efficiency. The book begins with the basic elements of convex sets and functions, and then describes various classes of convex optimization problems. Duality and approximation techniques are then covered, as are statistical estimation techniques. Various geometrical problems are then presented, and there is detailed discussion of unconstrained and constrained minimization problems, and interior-point methods. The focus of the book is on recognizing convex optimization problems and then finding the most appropriate technique for solving them. It contains many worked examples and homework exercises and will appeal to students, researchers and practitioners in fields such as engineering, computer science, mathematics, statistics, finance and economics.

rockafellar convex analysis: *Convex Optimization Theory* Dimitri Bertsekas, 2009-06-01 An insightful, concise, and rigorous treatment of the basic theory of convex sets and functions in finite dimensions, and the analytical/geometrical foundations of convex optimization and duality theory. Convexity theory is first developed in a simple accessible manner, using easily visualized proofs. Then the focus shifts to a transparent geometrical line of analysis to develop the fundamental duality between descriptions of convex functions in terms of points, and in terms of hyperplanes. Finally, convexity theory and abstract duality are applied to problems of constrained optimization, Fenchel and conic duality, and game theory to develop the sharpest possible duality results within a highly visual geometric framework. This on-line version of the book, includes an extensive set of theoretical problems with detailed high-quality solutions, which significantly extend the range and value of the book. The book may be used as a text for a theoretical convex optimization course; the author has taught several variants of such a course at MIT and elsewhere over the last ten years. It may also be used as a supplementary source for nonlinear programming classes, and as a theoretical foundation for classes focused on convex optimization models (rather than theory). It is an excellent supplement to several of our books: *Convex Optimization Algorithms* (Athena Scientific, 2015), *Nonlinear Programming* (Athena Scientific, 2017), *Network Optimization* (Athena Scientific, 1998), *Introduction to Linear Optimization* (Athena Scientific, 1997), and *Network Flows and Monotropic Optimization* (Athena Scientific, 1998).

rockafellar convex analysis: *Convex Analysis and Variational Problems* Ivar Ekeland, Roger Temam, 1999-12-01 This book contains different developments of infinite dimensional convex programming in the context of convex analysis, including duality, minmax and Lagrangians, and convexification of nonconvex optimization problems in the calculus of variations (infinite dimension). It also includes the theory of convex duality applied to partial differential equations; no other reference presents this in a systematic way. The minmax theorems contained in this book have many useful applications, in particular the robust control of partial differential equations in finite time horizon. First published in English in 1976, this SIAM Classics in Applied Mathematics edition contains the original text along with a new preface and some additional references.

rockafellar convex analysis: *Variational Analysis* R. Tyrrell Rockafellar, Roger J.-B. Wets, 2009-06-26 From its origins in the minimization of integral functionals, the notion of variations has evolved greatly in connection with applications in optimization, equilibrium, and control. This book develops a unified framework and provides a detailed exposition of variational geometry and

subdifferential calculus in their current forms beyond classical and convex analysis. Also covered are set-convergence, set-valued mappings, epi-convergence, duality, and normal integrands.

rockafellar convex analysis: An Easy Path to Convex Analysis and Applications Boris S. Mordukhovich, Nguyen Mau Nam, 2013-12-01 Convex optimization has an increasing impact on many areas of mathematics, applied sciences, and practical applications. It is now being taught at many universities and being used by researchers of different fields. As convex analysis is the mathematical f

rockafellar convex analysis: Convex Analysis and Nonlinear Optimization Jonathan Borwein, Adrian S. Lewis, 2010-05-05 Optimization is a rich and thriving mathematical discipline, and the underlying theory of current computational optimization techniques grows ever more sophisticated. This book aims to provide a concise, accessible account of convex analysis and its applications and extensions, for a broad audience. Each section concludes with an often extensive set of optional exercises. This new edition adds material on semismooth optimization, as well as several new proofs.

rockafellar convex analysis: Conjugate Duality and Optimization R. Tyrrell Rockafellar, 1974-01-01 Provides a relatively brief introduction to conjugate duality in both finite- and infinite-dimensional problems. An emphasis is placed on the fundamental importance of the concepts of Lagrangian function, saddle-point, and saddle-value. General examples are drawn from nonlinear programming, approximation, stochastic programming, the calculus of variations, and optimal control.

rockafellar convex analysis: Conjugate Duality in Convex Optimization Radu Ioan-Bot, 2010-02-04 The results presented in this book originate from the last decade research work of the author in the field of duality theory in convex optimization. The reputation of duality in the optimization theory comes mainly from the major role that it plays in formulating necessary and sufficient optimality conditions and, consequently, in generating different algorithmic approaches for solving mathematical programming problems. The investigations made in this work prove the importance of the duality theory beyond these aspects and emphasize its strong connections with different topics in convex analysis, nonlinear analysis, functional analysis and in the theory of monotone operators. The first part of the book brings to the attention of the reader the perturbation approach as a fundamental tool for developing the so-called conjugate duality theory. The classical Lagrange and Fenchel duality approaches are particular instances of this general concept. More than that, the generalized interior point regularity conditions stated in the past for the two mentioned situations turn out to be particularizations of the ones given in this general setting. In our investigations, the perturbation approach represents the starting point for deriving new duality concepts for several classes of convex optimization problems. Moreover, via this approach, generalized Moreau–Rockafellar formulae are provided and, in connection with them, a new class of regularity conditions, called closedness-type conditions, for both stable strong duality and strong duality is introduced. By stable strong duality we understand the situation in which strong duality still holds whenever perturbing the objective function of the primal problem with a linear continuous functional.

rockafellar convex analysis: Convexity and Duality in Optimization Jacob Ponstein, 2012-12-06 The analysis and optimization of convex functions have received a great deal of attention during the last two decades. If we had to choose two key-words from these developments, we would retain the concept of subdifferential and the duality theory. As it is usual in the development of mathematical theories, people have since tried to extend the known definitions and properties to new classes of functions, including the convex ones. For what concerns the generalization of the notion of subdifferential, tremendous achievements have been carried out in the past decade and any mathematician who is faced with a nondifferentiable nonconvex function has now a panoply of generalized subdifferentials or derivatives at his disposal. A lot remains to be done in this area, especially concerning vector-valued functions; however we think the golden age for these researches is behind us. Duality theory has also fascinated many mathematicians since the underlying mathematical framework has been laid down in the context of Convex Analysis. The

various duality schemes which have emerged in the recent years, despite of their mathematical elegance, have not always proved as powerful as expected.

rockafellar convex analysis: Variational Analysis R. Tyrrell Rockafellar, Roger J.-B. Wets, 2009-07-17 From its origins in the minimization of integral functionals, the notion of variations has evolved greatly in connection with applications in optimization, equilibrium, and control. This book develops a unified framework and provides a detailed exposition of variational geometry and subdifferential calculus in their current forms beyond classical and convex analysis. Also covered are set-convergence, set-valued mappings, epi-convergence, duality, and normal integrands.

rockafellar convex analysis: Statistical Inference Via Convex Optimization Anatoli Juditsky, Arkadi Nemirovski, 2020-04-07 This authoritative book draws on the latest research to explore the interplay of high-dimensional statistics with optimization. Through an accessible analysis of fundamental problems of hypothesis testing and signal recovery, Anatoli Juditsky and Arkadi Nemirovski show how convex optimization theory can be used to devise and analyze near-optimal statistical inferences. *Statistical Inference via Convex Optimization* is an essential resource for optimization specialists who are new to statistics and its applications, and for data scientists who want to improve their optimization methods. Juditsky and Nemirovski provide the first systematic treatment of the statistical techniques that have arisen from advances in the theory of optimization. They focus on four well-known statistical problems—sparse recovery, hypothesis testing, and recovery from indirect observations of both signals and functions of signals—demonstrating how they can be solved more efficiently as convex optimization problems. The emphasis throughout is on achieving the best possible statistical performance. The construction of inference routines and the quantification of their statistical performance are given by efficient computation rather than by analytical derivation typical of more conventional statistical approaches. In addition to being computation-friendly, the methods described in this book enable practitioners to handle numerous situations too difficult for closed analytical form analysis, such as composite hypothesis testing and signal recovery in inverse problems. *Statistical Inference via Convex Optimization* features exercises with solutions along with extensive appendixes, making it ideal for use as a graduate text.

rockafellar convex analysis: Convex Functions Jonathan M. Borwein, Jon D. Vanderwerff, 2010-01-14 The product of a collaboration of over 15 years, this volume is unique because it focuses on convex functions themselves, rather than on convex analysis. The authors explore the various classes and their characteristics, treating convex functions in both Euclidean and Banach spaces.

rockafellar convex analysis: Fundamentals of Convex Analysis Jean-Baptiste Hiriart-Urruty, Claude Lemaréchal, 2012-12-06 This book is an abridged version of the two volumes *Convex Analysis and Minimization Algorithms I and II* (Grundlehren der mathematischen Wissenschaften Vol. 305 and 306). It presents an introduction to the basic concepts in convex analysis and a study of convex minimization problems (with an emphasis on numerical algorithms). The backbone of both volumes was extracted, some material deleted which was deemed too advanced for an introduction, or too closely attached to numerical algorithms. Some exercises were included and finally the index has been considerably enriched, making it an excellent choice for the purpose of learning and teaching.

rockafellar convex analysis: Convex Analysis and Beyond Boris S. Mordukhovich, Nguyen Mau Nam, 2022-04-24 This book presents a unified theory of convex functions, sets, and set-valued mappings in topological vector spaces with its specifications to locally convex, Banach and finite-dimensional settings. These developments and expositions are based on the powerful geometric approach of variational analysis, which resides on set extremality with its characterizations and specifications in the presence of convexity. Using this approach, the text consolidates the device of fundamental facts of generalized differential calculus to obtain novel results for convex sets, functions, and set-valued mappings in finite and infinite dimensions. It also explores topics beyond convexity using the fundamental machinery of convex analysis to develop nonconvex generalized differentiation and its applications. The text utilizes an adaptable framework designed with researchers as well as multiple levels of students in mind. It includes many exercises

and figures suited to graduate classes in mathematical sciences that are also accessible to advanced students in economics, engineering, and other applications. In addition, it includes chapters on convex analysis and optimization in finite-dimensional spaces that will be useful to upper undergraduate students, whereas the work as a whole provides an ample resource to mathematicians and applied scientists, particularly experts in convex and variational analysis, optimization, and their applications.

rockafellar convex analysis: *Opt Art* Robert Bosch (mathématicien), 2019-11-12 Bosch provides a lively and accessible introduction to the geometric, algebraic, and algorithmic foundations of optimization. He presents classical applications, such as the legendary Traveling Salesman Problem, and shows how to adapt them to make optimization art-opt art. art.

rockafellar convex analysis: *Convex Optimization Algorithms* Dimitri Bertsekas, 2015-02-01 This book provides a comprehensive and accessible presentation of algorithms for solving convex optimization problems. It relies on rigorous mathematical analysis, but also aims at an intuitive exposition that makes use of visualization where possible. This is facilitated by the extensive use of analytical and algorithmic concepts of duality, which by nature lend themselves to geometrical interpretation. The book places particular emphasis on modern developments, and their widespread applications in fields such as large-scale resource allocation problems, signal processing, and machine learning. The book is aimed at students, researchers, and practitioners, roughly at the first year graduate level. It is similar in style to the author's 2009 *Convex Optimization Theory* book, but can be read independently. The latter book focuses on convexity theory and optimization duality, while the present book focuses on algorithmic issues. The two books share notation, and together cover the entire finite-dimensional convex optimization methodology. To facilitate readability, the statements of definitions and results of the theory book are reproduced without proofs in Appendix B.

rockafellar convex analysis: Discrete Convex Analysis Kazuo Murota, 2003-01-01 *Discrete Convex Analysis* is a novel paradigm for discrete optimization that combines the ideas in continuous optimization (convex analysis) and combinatorial optimization (matroid/submodular function theory) to establish a unified theoretical framework for nonlinear discrete optimization. The study of this theory is expanding with the development of efficient algorithms and applications to a number of diverse disciplines like matrix theory, operations research, and economics. This self-contained book is designed to provide a novel insight into optimization on discrete structures and should reveal unexpected links among different disciplines. It is the first and only English-language monograph on the theory and applications of discrete convex analysis.

rockafellar convex analysis: Generalized Convexity, Generalized Monotonicity: Recent Results Jean-Pierre Crouzeix, Juan Enrique Martinez Legaz, Michel Volle, 2013-12-01 A function is convex if its epigraph is convex. This geometrical structure has very strong implications in terms of continuity and differentiability. Separation theorems lead to optimality conditions and duality for convex problems. A function is quasiconvex if its lower level sets are convex. Here again, the geometrical structure of the level sets implies some continuity and differentiability properties for quasiconvex functions. Optimality conditions and duality can be derived for optimization problems involving such functions as well. Over a period of about fifty years, quasiconvex and other generalized convex functions have been considered in a variety of fields including economics, management science, engineering, probability and applied sciences in accordance with the need of particular applications. During the last twenty-five years, an increase of research activities in this field has been witnessed. More recently generalized monotonicity of maps has been studied. It relates to generalized convexity of functions as monotonicity relates to convexity. Generalized monotonicity plays a role in variational inequality problems, complementarity problems and more generally, in equilibrium problems.

rockafellar convex analysis: Nonsmooth Optimization and Related Topics F.H. Clarke, Vladimir F. Dem'yanov, F. Giannessi, 2013-11-11 This volume contains the edited texts of the lectures presented at the International School of Mathematics devoted to Nonsmooth Optimization, held from June 20 to July 1, 1988. The site for the meeting was the Ettore Majorana Centre for Scientific Research.

entific Culture in Erice, Sicily. In the tradition of these meetings the main purpose was to give the state-of-the-art of an important and growing field of mathematics, and to stimulate interactions between finite-dimensional and infinite-dimensional optimization. The School was attended by approximately 80 people from 23 countries; in particular it was possible to have some distinguished lecturers from the Soviet Union, whose research institutions are here gratefully acknowledged. Besides the lectures, several seminars were delivered; a special session was devoted to numerical computing aspects. The result was a broad exposure, giving a deep knowledge of the present research tendencies in the field. We wish to express our appreciation to all the participants. Special mention should be made of the Ettore Majorana Centre in Erice, which helped provide a stimulating and rewarding experience, and of its staff which was fundamental for the success of the meeting. Moreover, WP want to extend our deep appreci

rockafellar convex analysis: Advances in Convex Analysis and Global Optimization

Nicolas Hadjisavvas, Panos M. Pardalos, 2013-12-01 There has been much recent progress in global optimization algorithms for nonconvex continuous and discrete problems from both a theoretical and a practical perspective. Convex analysis plays a fundamental role in the analysis and development of global optimization algorithms. This is due essentially to the fact that virtually all nonconvex optimization problems can be described using differences of convex functions and differences of convex sets. A conference on Convex Analysis and Global Optimization was held during June 5 -9, 2000 at Pythagorion, Samos, Greece. The conference was honoring the memory of C. Caratheodory (1873-1950) and was endorsed by the Mathematical Programming Society (MPS) and by the Society for Industrial and Applied Mathematics (SIAM) Activity Group in Optimization. The conference was sponsored by the European Union (through the EPEAEK program), the Department of Mathematics of the Aegean University and the Center for Applied Optimization of the University of Florida, by the General Secretariat of Research and Technology of Greece, by the Ministry of Education of Greece, and several local Greek government agencies and companies. This volume contains a selective collection of refereed papers based on invited and contributing talks presented at this conference. The two themes of convexity and global optimization pervade this book. The conference provided a forum for researchers working on different aspects of convexity and global optimization to present their recent discoveries, and to interact with people working on complementary aspects of mathematical programming.

rockafellar convex analysis: Convex Optimization Sébastien Bubeck, 2015-11-12 This monograph presents the main complexity theorems in convex optimization and their corresponding algorithms. It begins with the fundamental theory of black-box optimization and proceeds to guide the reader through recent advances in structural optimization and stochastic optimization. The presentation of black-box optimization, strongly influenced by the seminal book by Nesterov, includes the analysis of cutting plane methods, as well as (accelerated) gradient descent schemes. Special attention is also given to non-Euclidean settings (relevant algorithms include Frank-Wolfe, mirror descent, and dual averaging), and discussing their relevance in machine learning. The text provides a gentle introduction to structural optimization with FISTA (to optimize a sum of a smooth and a simple non-smooth term), saddle-point mirror prox (Nemirovski's alternative to Nesterov's smoothing), and a concise description of interior point methods. In stochastic optimization it discusses stochastic gradient descent, mini-batches, random coordinate descent, and sublinear algorithms. It also briefly touches upon convex relaxation of combinatorial problems and the use of randomness to round solutions, as well as random walks based methods.

rockafellar convex analysis: *Interior-point Polynomial Algorithms in Convex Programming*

Yurii Nesterov, Arkadii Nemirovskii, 1994-01-01 Specialists working in the areas of optimization, mathematical programming, or control theory will find this book invaluable for studying interior-point methods for linear and quadratic programming, polynomial-time methods for nonlinear convex programming, and efficient computational methods for control problems and variational inequalities. A background in linear algebra and mathematical programming is necessary to understand the book. The detailed proofs and lack of numerical examples might suggest that the

book is of limited value to the reader interested in the practical aspects of convex optimization, but nothing could be further from the truth. An entire chapter is devoted to potential reduction methods precisely because of their great efficiency in practice.

rockafellar convex analysis: Convex Analysis Jan van Tiel, 1984 An introductory text on convex sets, convex functions and convex optimization. Emphasizes the basic concepts and the characteristic methods of convex mathematics, and includes proofs and theorems that focus on practical applications.

rockafellar convex analysis: From Analysis to Visualization David H. Bailey, Naomi Simone Borwein, Richard P. Brent, Regina S. Burachik, Judy-anne Heather Osborn, Brailey Sims, Qiji J. Zhu, 2020-03-16 Students and researchers from all fields of mathematics are invited to read and treasure this special Proceedings. A conference was held 25 -29 September 2017 at Noah's On the Beach, Newcastle, Australia, to commemorate the life and work of Jonathan M. Borwein, a mathematician extraordinaire whose untimely passing in August 2016 was a sorry loss to mathematics and to so many members of its community, a loss that continues to be keenly felt. A polymath, Jonathan Borwein ranks among the most wide ranging and influential mathematicians of the last 50 years, making significant contributions to an exceptional diversity of areas and substantially expanding the use of the computer as a tool of the research mathematician. The contributions in this commemorative volume probe Dr. Borwein's ongoing legacy in areas where he did some of his most outstanding work: Applied Analysis, Optimization and Convex Functions; Mathematics Education; Financial Mathematics; plus Number Theory, Special Functions and Pi, all tinged by the double prisms of Experimental Mathematics and Visualization, methodologies he championed.

rockafellar convex analysis: Delay-Adaptive Linear Control Yang Zhu, Miroslav Krstic, 2020-04-28 Basic predictor feedback for single-input systems -- Basic idea of adaptive control for single-input systems -- Single-input systems with full relative degree -- Single-input systems with arbitrary relative degree -- Exact predictor feedback for multi-input systems -- Full-state feedback of uncertain multi-input systems -- Output feedback of uncertain multi-input systems -- Output feedback of systems with uncertain delays, parameters and ODE state -- Predictor feedback for uncertainty-free systems -- Predictor feedback of uncertain single-input systems -- Predictor feedback of uncertain multi-input systems.

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mathematics and computer science.

rockafellar convex analysis: Introductory Lectures on Convex Optimization Y. Nesterov, 2013-12-01 It was in the middle of the 1980s, when the seminal paper by Kar markar opened a new epoch in nonlinear optimization. The importance of this paper, containing a new polynomial-time algorithm for linear op timization problems, was not only in its complexity bound. At that time, the most surprising feature of this algorithm was that the theoretical pre diction of its high efficiency was supported by excellent computational results. This unusual fact dramatically changed the style and direc tions of the research in nonlinear optimization. Thereafter it became more and more common that the new methods were provided with a complexity analysis, which was considered a better justification of their efficiency than computational experiments. In a new rapidly develop ing field, which got the name polynomial-time interior-point methods, such a justification was obligatory. Afteralmost fifteen years of intensive research, the main results of this development started to appear in monographs [12, 14, 16, 17, 18, 19]. Approximately at that time the author was asked to prepare a new course on nonlinear optimization for graduate students. The idea was to create a course which would reflect the new developments in the field. Actually, this was a major challenge. At the time only the theory of interior-point methods for linear optimization was polished enough to be explained to students. The general theory of self-concordant functions had appeared in print only once in the form of research monograph [12].

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resource for students and faculty alike.

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