NONCOMMUTATIVE ALGEBRAIC TOPOLOGY

Delving into the Intriguing World of Noncommutative Algebraic Topology

Introduction:

Are you fascinated by the intricate dance between algebra and geometry? Do concepts like spaces, mappings, and algebraic structures ignite your curiosity? Then prepare to embark on a journey into the fascinating realm of noncommutative algebraic topology, a field that pushes the boundaries of traditional topological understanding. This comprehensive guide will unravel the complexities of this advanced topic, providing a clear and accessible explanation of its core concepts, key developments, and future directions. We'll explore its fundamental principles, examine significant breakthroughs, and highlight the ongoing research shaping this dynamic area of mathematics. Get ready to unravel the mysteries of spaces that defy the commutative nature of classical topology.

What is Noncommutative Algebraic Topology?

Classical algebraic topology utilizes commutative algebras – like the cohomology ring of a topological space – to study topological properties. However, many important spaces and phenomena exhibit inherently noncommutative behavior. This is where noncommutative algebraic topology steps in. It provides a framework to analyze spaces and their properties using noncommutative algebraic structures, such as noncommutative rings, algebras, and categories. Instead of relying on the commutative structure of, say, cohomology rings, this field explores alternative algebraic invariants that capture the noncommutative aspects of the underlying topological spaces. Think of it as extending the toolbox of algebraic topology to handle more complex and nuanced situations.

Key Concepts and Techniques:

Noncommutative Geometry: This branch of mathematics provides the foundational language for noncommutative algebraic topology. It replaces the concept of a "point" in a space with a noncommutative algebra, effectively encoding the space's structure within algebraic relations. This allows for the study of spaces that lack a classical, commutative description.

Quantum Groups and Hopf Algebras: These algebraic structures play a crucial role in representing symmetries and deformations of spaces. Quantum groups, in particular, are noncommutative generalizations of Lie groups, enabling the analysis of spaces with quantum-mechanical properties or exhibiting non-classical symmetries.

Categorical Methods: Category theory provides a powerful language to describe and manipulate algebraic structures. In noncommutative algebraic topology, categories are used to organize and study various noncommutative invariants, offering a unified perspective on different approaches. This allows for a more abstract and flexible analysis of topological properties.

Cyclic Homology and Hochschild Homology: These are noncommutative generalizations of classical homology theories. They provide algebraic invariants that reflect the noncommutative structure of the underlying space, offering valuable tools for distinguishing noncommutative spaces.

Noncommutative Localization: This technique allows us to extend the scope of analysis by "inverting" elements in a noncommutative algebra, revealing previously inaccessible information about the space.

Historical Development and Significant Breakthroughs:

The foundations of noncommutative algebraic topology were laid in the late 20th century, spurred by advancements in noncommutative geometry and the recognition that many physically relevant systems exhibit noncommutative characteristics. Key breakthroughs included the development of cyclic homology, the application of quantum group theory to topological spaces, and the increasing use of categorical methods to unify different approaches. The field continues to evolve rapidly, driven by advancements in both pure mathematics and its applications in physics and other areas.

Applications and Current Research:

Noncommutative algebraic topology finds applications in various fields, including:

Quantum Physics: The study of quantum systems and their symmetries often necessitates the use of noncommutative techniques. Noncommutative spaces naturally arise in quantum mechanics, providing a mathematical framework to analyze these systems.

String Theory and Quantum Gravity: Noncommutative geometry and topology play a role in understanding the fundamental structures of spacetime at the Planck scale, providing potential insights into quantum gravity.

Condensed Matter Physics: The study of topological insulators and other exotic materials often requires the use of noncommutative methods to analyze their topological properties.

Computer Science: Noncommutative methods are finding applications in the study of quantum computation and the development of new algorithms.

Current research focuses on exploring new noncommutative invariants, developing more powerful computational techniques, and further investigating the connections between noncommutative topology and other areas of mathematics and physics.

A Hypothetical Textbook Outline: "Navigating Noncommutative Algebraic Topology"

I. Introduction:
What is Noncommutative Algebraic Topology?
Motivation and Historical Context
Overview of Key Concepts

II. Foundational Algebraic Structures:
Noncommutative Rings and Algebras
Hopf Algebras and Quantum Groups
Categorical Foundations

III. Noncommutative Homology Theories:
Cyclic Homology
Hochschild Homology
Derived Categories and Localization

IV. Applications in Geometry and Physics:
Noncommutative Geometry
Quantum Field Theory
Condensed Matter Physics

V. Advanced Topics and Open Problems:
Higher Categorical Structures
Homological Algebra and Noncommutative Spaces
Future Directions and Unsolved Questions

VI. Conclusion:
Summary of Key Findings
Future Research Directions
Impact and Significance

(Detailed Explanation of each point in the outline would comprise several hundred words each, elaborating on the mathematical concepts, providing examples, and showcasing relevant theorems and proofs. Due to space constraints, this detailed elaboration is omitted here. A full textbook would be required for such a detailed explanation.)

Frequently Asked Questions (FAQs):

1. What is the difference between commutative and noncommutative algebraic topology? Commutative algebraic topology uses commutative algebraic structures to study spaces, while noncommutative algebraic topology uses noncommutative structures, extending the analysis to spaces exhibiting noncommutative properties.

1. Why is noncommutative algebraic topology important? It provides tools to analyze spaces and systems that classical methods cannot handle, particularly those with noncommutative symmetries or quantum mechanical properties.

1. What are some examples of noncommutative spaces? Quantum spaces, spaces with non-classical symmetries, and certain spaces arising in string theory are examples.

1. What mathematical background is needed to study noncommutative algebraic topology? A strong foundation in algebra (abstract algebra, homological algebra), topology, and ideally category theory is essential.

1. What are the current research directions in this field? Current research focuses on developing new invariants, improving computational methods, and exploring connections to other fields like physics and computer science.

1. How does noncommutative algebraic topology relate to physics? It provides mathematical tools for analyzing quantum systems, quantum field theory, and aspects of string theory and quantum gravity.

1. What are some of the challenges in studying noncommutative algebraic topology? The complexity of noncommutative algebra and the abstract nature of the concepts make it a challenging field.

1. Are there any software packages or tools for computations in noncommutative algebraic topology? While there aren't dedicated packages like in some other areas, computational algebra systems can be used for certain computations.

1. Where can I find more resources to learn about noncommutative algebraic topology? Research papers, advanced textbooks on algebraic topology and noncommutative geometry, and university courses are good starting points.

Related Articles:

1. Noncommutative Geometry and its Applications: An overview of noncommutative geometry and its connections to various fields.

1. Cyclic Homology: A Primer: A beginner-friendly introduction to cyclic homology and its properties.

1. Hopf Algebras and Quantum Groups: An exploration of these algebraic structures and their significance in noncommutative topology.

1. Categorical Approaches to Noncommutative Algebra: The role of category theory in unifying and extending noncommutative algebraic methods.

1. Noncommutative Localization Techniques: An explanation of the technique of noncommutative localization and its applications.

1. Noncommutative Algebraic Topology and Quantum Field Theory: The interplay between these two fields and their mutual influence.

1. Recent Advances in Noncommutative Algebraic Topology: A survey of recent breakthroughs and current research directions.

1. Computational Aspects of Noncommutative Algebraic Topology: The use of computational methods to analyze noncommutative spaces.

1. Open Problems in Noncommutative Algebraic Topology: A discussion of outstanding questions and challenges in the field.

noncommutative algebraic topology: *Noncommutative Localization in Algebra and Topology* Andrew Ranicki, 2006-02-09 Noncommutative localization is a powerful algebraic technique for constructing new rings by inverting elements, matrices and more generally morphisms of modules. Originally conceived by algebraists (notably P. M. Cohn), it is now an important tool not only in pure algebra but also in the topology of non-simply-connected spaces, algebraic geometry and noncommutative geometry. This volume consists of 9 articles on noncommutative localization in algebra and topology by J. A. Beachy, P. M. Cohn, W. G. Dwyer, P. A. Linnell, A. Neeman, A. A. Ranicki, H. Reich, D. Sheiham and Z. Skoda. The articles include basic definitions, surveys, historical background and applications, as well as presenting new results. The book is an introduction to the subject, an account of the state of the art, and also provides many references for further material. It is suitable for graduate students and more advanced researchers in both algebra and topology.

noncommutative algebraic topology: Noncommutative Algebra Benson Farb, R. Keith Dennis, 2012-12-06 About This Book This book is meant to be used by beginning graduate students. It covers basic material needed by any student of algebra, and is essential to those specializing in ring theory, homological algebra, representation theory and K-theory, among others. It will also be of interest to students of algebraic topology, functional analysis, differential geometry and number theory. Our approach is more homological than ring-theoretic, as this leads the to many important areas of mathematics. This ap student more quickly proach is also, we believe, cleaner and easier to understand. However, the more classical, ring-theoretic approach, as well as modern extensions, are also presented via several exercises and sections in Chapter Five. We have tried not to leave any gaps on the paths to proving the main theorem- at most we ask the reader to fill in details for some of the sideline results; indeed this can be a fruitful way of solidifying one's understanding.

noncommutative algebraic topology: Noncommutative Geometry Alain Connes, Joachim Cuntz, Erik G. Guentner, Nigel Higson, Jerome Kaminker, John E. Roberts, 2003-12-15 Noncommutative Geometry is one of the most deep and vital research subjects of present-day Mathematics. Its development, mainly due to Alain Connes, is providing an increasing number of applications and deeper insights for instance in Foliations, K-Theory, Index Theory, Number Theory but also in Quantum Physics of elementary particles. The purpose of the Summer School in Martina Franca was to offer a fresh invitation to the subject and closely related topics; the contributions in this volume include the four main lectures, cover advanced developments and are delivered by prominent specialists.

noncommutative algebraic topology: Noncommutative Algebraic Geometry Gwyn Bellamy, Daniel Rogalski, Travis Schedler, J. Toby Stafford, Michael Wemyss, 2016-06-20 This book provides a comprehensive introduction to the interactions between noncommutative algebra and classical algebraic geometry.

noncommutative algebraic topology: Basic Noncommutative Geometry Masoud Khalkhali, 2009 Basic Noncommutative Geometry provides an introduction to noncommutative geometry and some of its applications. The book can be used either as a textbook for a graduate course on the subject or for self-study. It will be useful for graduate students and researchers in mathematics and theoretical physics and all those who are interested in gaining an understanding of the subject. One feature of this book is the wealth of examples and exercises that help the reader to navigate through the subject. While background material is provided in the text and in several appendices, some familiarity with basic notions of functional analysis, algebraic topology, differential geometry and homological algebra at a first year graduate level is helpful. Developed by Alain Connes since the late 1970s, noncommutative geometry has found many applications to long-standing conjectures in topology and geometry and has recently made headways in theoretical physics and number theory. The book starts with a detailed description of some of the most pertinent algebra-geometry correspondences by casting geometric notions in algebraic terms, then proceeds in the second chapter to the idea of a noncommutative space and how it is constructed. The last two chapters deal with homological tools: cyclic cohomology and Connes-Chern characters in K-theory and K-homology, culminating in one commutative diagram expressing the equality of topological and analytic index in a noncommutative setting. Applications to integrality of noncommutative topological invariants are given as well.--Publisher's description.

noncommutative algebraic topology: Non-commutative Algebraic Geometry F.M.J. van Oystaeyen, A.H.M.J. Verschoren, 2006-11-14

noncommutative algebraic topology: Noncommutative Geometry Igor V. Nikolaev, 2017-11-07 This book covers the basics of noncommutative geometry (NCG) and its applications in topology, algebraic geometry, and number theory. The author takes up the practical side of NCG and its value for other areas of mathematics. A brief survey of the main parts of NCG with historical remarks, bibliography, and a list of exercises is included. The presentation is intended for graduate students and researchers with interests in NCG, but will also serve nonexperts in the field. Contents Part I: Basics Model examples Categories and functors C∗-algebras Part II: Noncommutative invariants Topology Algebraic geometry Number theory Part III: Brief survey of NCG Finite geometries Continuous geometries Connes geometries Index theory Jones polynomials Quantum groups Noncommutative algebraic geometry Trends in noncommutative geometry

noncommutative algebraic topology: K-theory and Noncommutative Geometry Guillermo Cortiñas, 2008 Since its inception 50 years ago, K-theory has been a tool for understanding a wide-ranging family of mathematical structures and their invariants: topological spaces, rings, algebraic varieties and operator algebras are the dominant examples. The invariants range from characteristic classes in cohomology, determinants of matrices, Chow groups of varieties, as well as traces and indices of elliptic operators. Thus K-theory is notable for its connections with other branches of mathematics. Noncommutative geometry develops tools which allow one to think of noncommutative algebras in the same footing as commutative ones: as algebras of functions on (noncommutative) spaces. The algebras in question come from problems in various areas of mathematics and mathematical physics; typical examples include algebras of pseudodifferential operators, group algebras, and other algebras arising from quantum field theory. To study noncommutative geometric problems one considers invariants of the relevant noncommutative algebras. These invariants include algebraic and topological K-theory, and also cyclic homology, discovered independently by Alain Connes and Boris Tsygan, which can be regarded both as a noncommutative version of de Rham cohomology and as an additive version of K-theory. There are primary and secondary Chern characters which pass from K-theory to cyclic homology. These characters are relevant both to noncommutative and commutative problems and have applications ranging from index theorems to the detection of singularities of commutative algebraic varieties. The contributions to this volume represent this range of connections between K-theory, noncommmutative geometry, and other branches of mathematics.

noncommutative algebraic topology: From Differential Geometry to Non-commutative Geometry and Topology Neculai S. Teleman, 2019-11-10 This book aims to provide a friendly introduction to non-commutative geometry. It studies index theory from a classical differential geometry perspective up to the point where classical differential geometry methods become insufficient. It then presents non-commutative geometry as a natural continuation of classical differential geometry. It thereby aims to provide a natural link between classical differential geometry and non-commutative geometry. The book shows that the index formula is a topological statement, and ends with non-commutative topology.

noncommutative algebraic topology: Noncommutative Geometry and Particle Physics Walter D. van Suijlekom, 2014-07-21 This book provides an introduction to noncommutative geometry and presents a number of its recent applications to particle physics. It is intended for graduate students in mathematics/theoretical physics who are new to the field of noncommutative geometry, as well as for researchers in mathematics/theoretical physics with an interest in the physical applications of noncommutative geometry. In the first part, we introduce the main concepts and techniques by studying finite noncommutative spaces, providing a “light” approach to noncommutative geometry. We then proceed with the general framework by defining and analyzing noncommutative spin manifolds and deriving some main results on them, such as the local index formula. In the second part, we show how noncommutative spin manifolds naturally give rise to gauge theories, applying this principle to specific examples. We subsequently geometrically derive abelian and non-abelian Yang-Mills gauge theories, and eventually the full Standard Model of particle physics, and conclude by explaining how noncommutative geometry might indicate how to proceed beyond the Standard Model.

noncommutative algebraic topology: Galois Theories Francis Borceux, George Janelidze, 2001-02-22 Starting from the classical finite-dimensional Galois theory of fields, this book develops Galois theory in a much more general context, presenting work by Grothendieck in terms of separable algebras and then proceeding to the infinite-dimensional case, which requires considering topological Galois groups. In the core of the book, the authors first formalize the categorical context in which a general Galois theorem holds, and then give applications to Galois theory for commutative rings, central extensions of groups, the topological theory of covering maps and a Galois theorem for toposes. The book is designed to be accessible to a wide audience: the prerequisites are first courses in algebra and general topology, together with some familiarity with the categorical notions of limit and adjoint functors. The first chapters are accessible to advanced undergraduates, with later ones at a graduate level. For all algebraists and category theorists this book will be a rewarding read.

noncommutative algebraic topology: Geometric Models for Noncommutative Algebras Ana Cannas da Silva, Alan Weinstein, 1999 The volume is based on a course, ``Geometric Models for Noncommutative Algebras'' taught by Professor Weinstein at Berkeley. Noncommutative geometry is the study of noncommutative algebras as if they were algebras of functions on spaces, for example, the commutative algebras associated to affine algebraic varieties, differentiable manifolds, topological spaces, and measure spaces. In this work, the authors discuss several types of geometric objects (in the usual sense of sets with structure) that are closely related to noncommutative algebras. Central to the discussion are symplectic and Poisson manifolds, which arise when noncommutative algebras are obtained by deforming commutative algebras. The authors also give a detailed study of groupoids (whose role in noncommutative geometry has been stressed by Connes) as well as of Lie algebroids, the infinitesimal approximations to differentiable groupoids. Featured are many interesting examples, applications, and exercises. The book starts with basic definitions and builds to (still) open questions. It is suitable for use as a graduate text. An extensive bibliography and index are included.

noncommutative algebraic topology: *Applications of Algebraic Topology* S. Lefschetz, 2012-12-06 This monograph is based, in part, upon lectures given in the Princeton School of Engineering and Applied Science. It presupposes mainly an elementary knowledge of linear algebra and of topology. In topology the limit is dimension two mainly in the latter chapters and questions of topological invariance are carefully avoided. From the technical viewpoint graphs is our only requirement. However, later, questions notably related to Kuratowski's classical theorem have demanded an easily provided treatment of 2-complexes and surfaces. January 1972 Solomon Lefschetz 4 INTRODUCTION The study of electrical networks rests upon preliminary theory of graphs. In the literature this theory has always been dealt with by special ad hoc methods. My purpose here is to show that actually this theory is nothing else than the first chapter of classical algebraic topology and may be very advantageously treated as such by the well known methods of that science. Part I of this volume covers the following ground: The first two chapters present, mainly in outline, the needed basic elements of linear algebra. In this part duality is dealt with somewhat more extensively. In Chapter III the merest elements of general topology are discussed. Graph theory proper is covered in Chapters IV and v, first structurally and then as algebra. Chapter VI discusses the applications to networks. In Chapters VII and VIII the elements of the theory of 2-dimensional complexes and surfaces are presented.

noncommutative algebraic topology: *Topics in Non-Commutative Geometry* Y. Manin, 2014-07-14 There is a well-known correspondence between the objects of algebra and geometry: a space gives rise to a function algebra; a vector bundle over the space corresponds to a projective module over this algebra; cohomology can be read off the de Rham complex; and so on. In this book

Yuri Manin addresses a variety of instances in which the application of commutative algebra cannot be used to describe geometric objects, emphasizing the recent upsurge of activity in studying noncommutative rings as if they were function rings on noncommutative spaces. Manin begins by summarizing and giving examples of some of the ideas that led to the new concepts of noncommutative geometry, such as Connes' noncommutative de Rham complex, supergeometry, and quantum groups. He then discusses supersymmetric algebraic curves that arose in connection with superstring theory; examines superhomogeneous spaces, their Schubert cells, and superanalogues of Weyl groups; and provides an introduction to quantum groups. This book is intended for mathematicians and physicists with some background in Lie groups and complex geometry. Originally published in 1991. The Princeton Legacy Library uses the latest print-on-demand technology to again make available previously out-of-print books from the distinguished backlist of Princeton University Press. These editions preserve the original texts of these important books while presenting them in durable paperback and hardcover editions. The goal of the Princeton Legacy Library is to vastly increase access to the rich scholarly heritage found in the thousands of books published by Princeton University Press since its founding in 1905.

noncommutative algebraic topology: Infinite Group Theory: From The Past To The Future Paul Baginski, Benjamin Fine, Anthony Gaglione, 2017-12-26 The development of algebraic geometry over groups, geometric group theory and group-based cryptography, has led to there being a tremendous recent interest in infinite group theory. This volume presents a good collection of papers detailing areas of current interest.

noncommutative algebraic topology: A Concise Course in Algebraic Topology J. P. May, 1999-09 Algebraic topology is a basic part of modern mathematics, and some knowledge of this area is indispensable for any advanced work relating to geometry, including topology itself, differential geometry, algebraic geometry, and Lie groups. This book provides a detailed treatment of algebraic topology both for teachers of the subject and for advanced graduate students in mathematics either specializing in this area or continuing on to other fields. J. Peter May's approach reflects the enormous internal developments within algebraic topology over the past several decades, most of which are largely unknown to mathematicians in other fields. But he also retains the classical presentations of various topics where appropriate. Most chapters end with problems that further explore and refine the concepts presented. The final four chapters provide sketches of substantial areas of algebraic topology that are normally omitted from introductory texts, and the book concludes with a list of suggested readings for those interested in delving further into the field.

noncommutative algebraic topology: *Virtual Topology and Functor Geometry* Fred Van Oystaeyen, 2007-11-15 Intrinsically noncommutative spaces today are considered from the perspective of several branches of modern physics, including quantum gravity, string theory, and statistical physics. From this point of view, it is ideal to devise a concept of space and its geometry that is fundamentally noncommutative. Providing a clear introduction to noncommutat

noncommutative algebraic topology: Elements of Noncommutative Geometry Jose M. Gracia-Bondia, Joseph C. Varilly, Hector Figueroa, 2013-11-27

noncommutative algebraic topology: Noncommutative Geometry and Cayley-smooth Orders Lieven Le Bruyn, 2007-08-24 Noncommutative Geometry and Cayley-smooth Orders explains the theory of Cayley-smooth orders in central simple algebras over function fields of varieties. In particular, the book describes the etale local structure of such orders as well as their central singularities and finite dimensional representations. After an introduction to partial d

noncommutative algebraic topology: *An Invitation To Noncommutative Geometry* Matilde Marcolli, Masoud Khalkhali, 2008-02-11 This is the first existing volume that collects lectures on this important and fast developing subject in mathematics. The lectures are given by leading experts in the field and the range of topics is kept as broad as possible by including both the algebraic and the differential aspects of noncommutative geometry as well as recent applications to theoretical physics and number theory.

noncommutative algebraic topology: Quantization, Geometry and Noncommutative

Structures in Mathematics and Physics Alexander Cardona, Pedro Morales, Hernán Ocampo, Sylvie Paycha, Andrés F. Reyes Lega, 2017-10-26 This monograph presents various ongoing approaches to the vast topic of quantization, which is the process of forming a quantum mechanical system starting from a classical one, and discusses their numerous fruitful interactions with mathematics.The opening chapter introduces the various forms of quantization and their interactions with each other and with mathematics.A first approach to quantization, called deformation quantization, consists of viewing the Planck constant as a small parameter. This approach provides a deformation of the structure of the algebra of classical observables rather than a radical change in the nature of the observables. When symmetries come into play, deformation quantization needs to be merged with group actions, which is presented in chapter 2, by Simone Gutt.The noncommutativity arising from quantization is the main concern of noncommutative geometry. Allowing for the presence of symmetries requires working with principal fiber bundles in a non-commutative setup, where Hopf algebras appear naturally. This is the topic of chapter 3, by Christian Kassel. Nichols algebras, a special type of Hopf algebras, are the subject of chapter 4, by Nicolás Andruskiewitsch. The purely algebraic approaches given in the previous chapters do not take the geometry of space-time into account. For this purpose a special treatment using a more geometric point of view is required. An approach to field quantization on curved space-time, with applications to cosmology, is presented in chapter 5 in an account of the lectures of Abhay Ashtekar that brings a complementary point of view to non-commutativity.An alternative quantization procedure is known under the name of string theory. In chapter 6 its supersymmetric version is presented. Superstrings have drawn the attention of many mathematicians, due to its various fruitful interactions with algebraic geometry, some of which are described here. The remaining chapters discuss further topics, as the Batalin-Vilkovisky formalism and direct products of spectral triples.This volume addresses both physicists and mathematicians and serves as an introduction to ongoing research in very active areas of mathematics and physics at the border line between geometry, topology, algebra and quantum field theory.

noncommutative algebraic topology: *Advances in Noncommutative Geometry* Ali Chamseddine, Caterina Consani, Nigel Higson, Masoud Khalkhali, Henri Moscovici, Guoliang Yu, 2020-01-13 This authoritative volume in honor of Alain Connes, the foremost architect of Noncommutative Geometry, presents the state-of-the art in the subject. The book features an amalgam of invited survey and research papers that will no doubt be accessed, read, and referred to, for several decades to come. The pertinence and potency of new concepts and methods are concretely illustrated in each contribution. Much of the content is a direct outgrowth of the Noncommutative Geometry conference, held March 23–April 7, 2017, in Shanghai, China. The conference covered the latest research and future areas of potential exploration surrounding topology and physics, number theory, as well as index theory and its ramifications in geometry.

noncommutative algebraic topology: Noncommutative Geometry, Quantum Fields and Motives Alain Connes, Matilde Marcolli, 2019-03-13 The unifying theme of this book is the interplay among noncommutative geometry, physics, and number theory. The two main objects of investigation are spaces where both the noncommutative and the motivic aspects come to play a role: space-time, where the guiding principle is the problem of developing a quantum theory of gravity, and the space of primes, where one can regard the Riemann Hypothesis as a long-standing problem motivating the development of new geometric tools. The book stresses the relevance of noncommutative geometry in dealing with these two spaces. The first part of the book deals with quantum field theory and the geometric structure of renormalization as a Riemann-Hilbert correspondence. It also presents a model of elementary particle physics based on noncommutative geometry. The main result is a complete derivation of the full Standard Model Lagrangian from a very simple mathematical input. Other topics covered in the first part of the book are a noncommutative geometry model of dimensional regularization and its role in anomaly computations, and a brief introduction to motives and their conjectural relation to quantum field theory. The second part of the book gives an interpretation of the Weil explicit formula as a trace formula and a spectral realization of the zeros of the Riemann zeta function. This is based on the noncommutative geometry of the adèle class space, which is also described as the space of commensurability classes of Q-lattices, and is dual to a noncommutative motive (endomotive) whose cyclic homology provides a general setting for spectral realizations of zeros of L-functions. The quantum statistical mechanics of the space of Q-lattices, in one and two dimensions, exhibits spontaneous symmetry breaking. In the low-temperature regime, the equilibrium states of the corresponding systems are related to points of classical moduli spaces and the symmetries to the class field theory of the field of rational numbers and of imaginary quadratic fields, as well as to the automorphisms of the field of modular functions. The book ends with a set of analogies between the noncommutative geometries underlying the mathematical formulation of the Standard Model minimally coupled to gravity and the moduli spaces of Q-lattices used in the study of the zeta function.

noncommutative algebraic topology: *Nonabelian Algebraic Topology* Ronald Brown, Philip J. Higgins, Rafael Sivera, 2011 The main theme of this book is that the use of filtered spaces rather than just topological spaces allows the development of basic algebraic topology in terms of higher homotopy groupoids; these algebraic structures better reflect the geometry of subdivision and composition than those commonly in use. Exploration of these uses of higher dimensional versions of groupoids has been largely the work of the first two authors since the mid 1960s. The structure of the book is intended to make it useful to a wide class of students and researchers for learning and evaluating these methods, primarily in algebraic topology but also in higher category theory and its applications in analogous areas of mathematics, physics, and computer science. Part I explains the intuitions and theory in dimensions 1 and 2, with many figures and diagrams, and a detailed account of the theory of crossed modules. Part II develops the applications of crossed complexes. The engine driving these applications is the work of Part III on cubical ω-groupoids, their relations to crossed complexes, and their homotopically defined examples for filtered spaces. Part III also includes a chapter suggesting further directions and problems, and three appendices give accounts of some relevant aspects of category theory. Endnotes for each chapter give further history and references.

noncommutative algebraic topology: A First Course in Noncommutative Rings T.Y. Lam, 2012-12-06 One of my favorite graduate courses at Berkeley is Math 251, a one-semester course in ring theory offered to second-year level graduate students. I taught this course in the Fall of 1983, and more recently in the Spring of 1990, both times focusing on the theory of noncommutative rings. This book is an outgrowth of my lectures in these two courses, and is intended for use by instructors and graduate students in a similar one-semester course in basic ring theory. Ring theory is a subject of central importance in algebra. Historically, some of the major discoveries in ring theory have helped shape the course of development of modern abstract algebra. Today, ring theory is a fer tile meeting ground for group theory (group rings), representation theory (modules), functional analysis (operator algebras), Lie theory (enveloping algebras), algebraic geometry (finitely generated algebras, differential op erators, invariant theory), arithmetic (orders, Brauer groups), universal algebra (varieties of rings), and homological algebra (cohomology of rings, projective modules, Grothendieck and higher K-groups). In view of these basic connections between ring theory and other branches of mathemat ics, it is perhaps no exaggeration to say that a course in ring theory is an indispensable part of the education for any fledgling algebraist. The purpose of my lectures was to give a general introduction to the theory of rings, building on what the students have learned from a stan dard first-year graduate course in abstract algebra.

noncommutative algebraic topology: *Algebraic Geometry for Associative Algebras* Freddy Van Oystaeyen, 2000-06-06 This work focuses on the association of methods from topology, category and sheaf theory, algebraic geometry, noncommutative and homological algebras, quantum groups and spaces, rings of differential operation, Cech and sheaf cohomology theories, and dimension theories to create a blend of noncommutative algebraic geometry. It offers a scheme theory that sustains the duality between algebraic geometry and commutative algebra to the noncommutative level.

noncommutative algebraic topology: Cyclic Homology Jean-Louis Loday, 2013-06-29 This book is a comprehensive study of cyclic homology theory together with its relationship with Hochschild homology, de Rham cohomology, S1 equivariant homology, the Chern character, Lie algebra homology, algebraic K-theory and non-commutative differential geometry. Though conceived as a basic reference on the subject, many parts of this book are accessible to graduate students.

noncommutative algebraic topology: An Introduction to Noncommutative Differential Geometry and Its Physical Applications J. Madore, 1999-06-24 A thoroughly revised introduction to non-commutative geometry.

noncommutative algebraic topology: Algebraic Topology Via Differential Geometry M. Karoubi, C. Leruste, 1987 In this volume the authors seek to illustrate how methods of differential geometry find application in the study of the topology of differential manifolds. Prerequisites are few since the authors take pains to set out the theory of differential forms and the algebra required. The reader is introduced to De Rham cohomology, and explicit and detailed calculations are present as examples. Topics covered include Mayer-Vietoris exact sequences, relative cohomology, Pioncare duality and Lefschetz's theorem. This book will be suitable for graduate students taking courses in algebraic topology and in differential topology. Mathematicians studying relativity and mathematical physics will find this an invaluable introduction to the techniques of differential geometry.

noncommutative algebraic topology: Arithmetic Noncommutative Geometry Matilde Marcolli, 2005 Arithmetic Noncommutative Geometry uses ideas and tools from noncommutative geometry to address questions in a new way and to reinterpret results and constructions from number theory and arithmetic algebraic geometry. This general philosophy is applied to the geometry and arithmetic of modular curves and to the fibers at Archimedean places of arithmetic surfaces and varieties. Noncommutative geometry can be expected to say something about topics of arithmetic interest because it provides the right framework for which the tools of geometry continue to make sense on spaces that are very singular and apparently very far from the world of algebraic varieties. This provides a way of refining the boundary structure of certain classes of spaces that arise in the context of arithmetic geometry. With a foreword written by Yuri Manin and a brief introduction to noncommutative geometry, this book offers a comprehensive account of the cross fertilization between two important areas, noncommutative geometry and number theory. It is suitable for graduate students and researchers interested in these areas.

noncommutative algebraic topology: *An Introduction to Noncommutative Spaces and Their Geometries* Giovanni Landi, 2003-07-01 These lecture notes are an introduction to several ideas and applications of noncommutative geometry. It starts with a not necessarily commutative but associative algebra which is thought of as the algebra of functions on some 'virtual noncommutative space'. Attention is switched from spaces, which in general do not even exist, to algebras of functions. In these notes, particular emphasis is put on seeing noncommutative spaces as concrete spaces, namely as a collection of points with a topology. The necessary mathematical tools are presented in a systematic and accessible way and include among other things, C'*-algebras, module theory and K-theory, spectral calculus, forms and connection theory. Application to Yang--Mills, fermionic, and gravity models are described. Also the spectral action and the related invariance under automorphism of the algebra is illustrated. Some recent work on noncommutative lattices is presented. These lattices arose as topologically nontrivial approximations to 'contuinuum' topological spaces. They have been used to construct quantum-mechanical and field-theory models, alternative models to lattice gauge theory, with nontrivial topological content. This book will be essential to physicists and mathematicians with an interest in noncommutative geometry and its uses in physics.

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