non associative algebra

Delving into the Intriguing World of Non-Associative Algebra

Introduction:

Are you fascinated by the intricate world of mathematics beyond the familiar rules of arithmetic? If you've ever wondered what happens when the associative law – that familiar property (ab)c = a(bc) – is abandoned, then prepare to dive into the captivating realm of non-associative algebra. This comprehensive guide will unravel the mysteries of this fascinating branch of mathematics, exploring its core concepts, key examples, and significant applications. We'll journey from foundational definitions to advanced applications, providing a robust understanding suitable for both curious beginners and seasoned mathematicians. Prepare to discover a world where the order of operations suddenly matters in a profoundly different way!

1. Understanding the Associative Law and its Absence

The associative law, a cornerstone of traditional algebra, dictates that the grouping of elements in a multiplication operation doesn't affect the result. However, in non-associative algebras, this crucial property is relinquished, opening up a vast landscape of mathematical structures with unique properties and behaviors. This seemingly small change leads to drastically different algebraic structures, demanding a re-evaluation of many fundamental algebraic concepts. We'll explore the implications of this lack of associativity and how it fundamentally alters our understanding of algebraic operations.

2. Key Examples of Non-Associative Algebras

Several important and widely studied non-associative algebraic structures exist. We'll explore some prominent examples:

Lie Algebras: These algebras play a crucial role in physics, particularly in the representation theory of Lie groups and in the study of symmetries. We'll delve into their defining properties, such as anti-commutativity ([x,y] = -[y,x]) and the Jacobi identity.

Jordan Algebras: These algebras, characterized by commutativity and a specific identity involving the Jordan product, find applications in quantum mechanics and the study of symmetric spaces. We'll examine their unique structure and their relationship to other algebraic systems.

Alternative Algebras: These algebras satisfy a weaker form of associativity,

known as alternativity, which holds for pairs of repeated elements ($a(ab) = a^2b$ and $(ab)b = ab^2$). This subtle difference introduces a rich mathematical landscape with its own set of interesting properties.

Octonions: These are an eight-dimensional non-associative algebra that extend the quaternions, showcasing a fascinating interplay between non-associativity and other algebraic properties. We'll explore their unique characteristics and their applications in various fields, including physics and computer science.

3. The Importance of the Order of Operations in Non-Associative Algebras

In contrast to associative algebras, the order in which operations are performed becomes paramount in non-associative algebras. The expression $(ab)c$ can yield a drastically different result from $a(bc)$. This necessitates careful notation and a deeper understanding of the underlying structure to avoid ambiguity and ensure accurate calculations. We'll explore specific examples to illustrate the dramatic impact of altering the order of operations.

4. Applications of Non-Associative Algebras

Contrary to their seemingly abstract nature, non-associative algebras have significant applications across multiple scientific disciplines:

Physics: Lie algebras underpin the study of symmetries in particle physics and provide a crucial framework for understanding gauge theories. Jordan algebras find applications in quantum mechanics.

Geometry: Non-associative algebras are instrumental in the study of certain types of geometries, particularly those related to non-Euclidean spaces.

Computer Science: Certain non-associative structures have found applications in areas such as cryptography and algorithm design.

5. Advanced Topics in Non-Associative Algebra (Brief Overview)

For those seeking a more in-depth exploration, we will briefly touch upon advanced concepts such as:

Identities and Representations: Understanding the defining identities and representations of different non-associative algebras.

Automorphisms and Derivations: Exploring the symmetries and transformations within these algebraic structures.

Structure Theory: Investigating the classification and decomposition of non-associative algebras.

Book Outline: "A Journey into Non-Associative Algebras"

I. Introduction:
What is Non-Associative Algebra?
The Importance of the Associative Law
Overview of Key Concepts and Applications

II. Fundamental Concepts:
Defining Non-Associative Algebras
Examples and Counterexamples
The Role of Order of Operations

III. Key Types of Non-Associative Algebras:
Lie Algebras: Definition, Properties, and Applications
Jordan Algebras: Definition, Properties, and Applications
Alternative Algebras: Definition, Properties, and Applications
Octonions: Definition, Properties, and Applications

IV. Applications and Further Exploration:
Applications in Physics
Applications in Geometry
Applications in Computer Science
Advanced Topics and Open Problems

V. Conclusion:
Summary of Key Findings
Directions for Future Research

Detailed Explanation of the Book Outline Points:

(Each point in the outline would require a substantial section, mirroring the content already established in the blog post above. Due to space constraints, only a brief elaboration is provided here. A full book would expand each point significantly.)

I. Introduction: This section would provide a comprehensive introduction to the field, defining non-associative algebras and highlighting their significance within mathematics and beyond.

II. Fundamental Concepts: This chapter lays the groundwork by formally defining non-associative algebras, providing illustrative examples, and emphasizing the crucial impact of the order of operations.

III. Key Types of Non-Associative Algebras: This would be the core of the

book, dedicating substantial chapters to each of the major classes of non-associative algebras (Lie, Jordan, Alternative, Octonions), explaining their defining properties, illustrating examples, and detailing their applications.

IV. Applications and Further Exploration: This section would delve into the practical applications of non-associative algebras across various scientific disciplines, paving the way for discussion of more advanced concepts and open research questions.

V. Conclusion: This concluding chapter would summarize the key concepts discussed throughout the book, offering a perspective on the current state of the field and pointing towards potential avenues for future research.

FAQs:

1. What is the difference between associative and non-associative algebras? Associative algebras satisfy the associative law ((ab)c = a(bc)), while non-associative algebras do not.

1. What are some real-world applications of non-associative algebras? They find applications in physics (Lie algebras in particle physics), geometry (certain types of geometries), and computer science (cryptography, algorithm design).

1. Are Lie algebras commutative? No, Lie algebras are non-commutative; the Lie bracket satisfies [x,y] = -[y,x].

1. What are octonions? Octonions are an eight-dimensional non-associative algebra that extends the quaternions.

1. What is the Jacobi identity? It's a crucial identity in Lie algebras relating the Lie bracket to associativity in a specific way.

1. What are Jordan algebras used for? They find applications in quantum mechanics and the study of symmetric spaces.

1. What is alternativity? It's a weaker form of associativity that holds for repeated elements ($a(ab) = a^2b$ and $(ab)b = ab^2$).

1. Are non-associative algebras useful in practical applications? Yes, they are crucial in various scientific fields, including physics, geometry, and computer science.

1. Where can I learn more about non-associative algebras? Advanced textbooks on abstract algebra and specialized publications focusing on specific types of non-associative algebras are excellent resources.

Related Articles:

1. Lie Algebras: A Beginner's Guide: An introductory overview of Lie algebras, their properties, and applications.

1. Jordan Algebras and Quantum Mechanics: Exploring the connection between Jordan algebras and their use in quantum mechanical systems.

1. The Exceptional Lie Algebras: A detailed examination of the exceptional Lie algebras, their unique properties, and their role in mathematical physics.

1. Alternative Algebras and Moufang Loops: Exploring the relationship between alternative algebras and Moufang loops.

1. Octonions and their Geometric Interpretations: A discussion of the geometric properties of octonions and their applications.

1. Non-Associative Algebras in String Theory: An exploration of the role of non-associative algebras in string theory and related areas of theoretical physics.

1. Applications of Non-Associative Algebras in Cryptography: A study of the use of non-associative algebraic structures in cryptographic systems.

1. The Structure Theory of Non-Associative Algebras: An overview of the classification and structural analysis of various types of non-associative algebras.

1. Open Problems in Non-Associative Algebra: A discussion of current research challenges and open questions in the field.

non associative algebra: Non-Associative Algebra and Its Applications Lev Sabinin, Larissa Sbitneva, Ivan Shestakov, 2006-01-13 With contributions derived from presentations at an international conference, Non-Associative Algebra and Its Applications explores a wide range of topics focusing on Lie algebras, nonassociative rings and algebras, quasigroups, loops, and related systems as well as applications of nonassociative algebra to geometry, physics, and natural sciences. This book covers material such as Jordan superalgebras, nonassociative deformations, nonassociative generalization of Hopf algebras, the structure of free algebras, derivations of Lie algebras, and the identities of Albert algebra. It also includes applications of smooth quasigroups and loops to differential geometry and relativity.

non associative algebra: Introduction to Octonion and Other Non-Associative Algebras in Physics Susumu Okubo, 1995-08-03 In this book, the author aims to familiarize researchers and graduate students in both physics and mathematics with the application of non-associative algebras in physics.Topics covered by the author range from algebras of observables in quantum mechanics, angular momentum and octonions, division algebra, triple-linear products and YangSHBaxter equations. The author also covers non-associative gauge theoretic reformulation of Einstein's general relativity theory and so on. Much of the material found in this book is not available in other standard works.

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non associative algebra: Non-Associative Algebra and Its Applications Santos González, 2012-12-06 This volume contains the proceedings of the Third International Conference on Non-Associative Algebra and Its Applications, held in Oviedo, Spain, July 12--17, 1993. The

conference brought together specialists from all over the world who work in this interesting and active field, which is currently enjoying much attention. All aspects of non-associative algebra are covered. Topics range from purely mathematical subjects to a wide spectrum of applications, and from state-of-the-art articles to overview papers. This collection will point the way for further research for many years to come. The volume is of interest to researchers in mathematics as well as those whose work involves the application of non-associative algebra in such areas as physics, biology and genetics.

non associative algebra: Associative and Non-Associative Algebras and Applications Mercedes Siles Molina, Laiachi El Kaoutit, Mohamed Louzari, L'Moufadal Ben Yakoub, Mohamed Benslimane, 2020-01-02 This book gathers together selected contributions presented at the 3rd Moroccan Andalusian Meeting on Algebras and their Applications, held in Chefchaouen, Morocco, April 12-14, 2018, and which reflects the mathematical collaboration between south European and north African countries, mainly France, Spain, Morocco, Tunisia and Senegal. The book is divided in three parts and features contributions from the following fields: algebraic and analytic methods in associative and non-associative structures; homological and categorical methods in algebra; and history of mathematics. Covering topics such as rings and algebras, representation theory, number theory, operator algebras, category theory, group theory and information theory, it opens up new avenues of study for graduate students and young researchers. The findings presented also appeal to anyone interested in the fields of algebra and mathematical analysis.

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non associative algebra: Non-Associative Algebras and Related Topics Helena Albuquerque, Jose Brox, Consuelo Martínez, Paulo Saraiva, 2023-07-28 This proceedings volume presents a selection of peer-reviewed contributions from the Second Non-Associative Algebras and Related Topics (NAART II) conference, which was held at the University of Coimbra, Portugal, from July 18–22, 2022. The conference was held in honor of mathematician Alberto Elduque, who has made significant contributions to the study of non-associative structures such as Lie, Jordan, and Leibniz algebras. The papers in this volume are organized into four parts: Lie algebras, superalgebras, and groups; Leibniz algebras; associative and Jordan algebras; and other non-associative structures. They cover a variety of topics, including classification problems, special maps (automorphisms, derivations, etc.), constructions that relate different structures, and representation theory. One of the unique features of NAART is that it is open to all topics related to non-associative algebras, including octonion algebras, composite algebras, Banach algebras, connections with geometry, applications in coding theory, combinatorial problems, and more. This diversity allows researchers from a range of fields to find the conference subjects interesting and discover connections with their own areas, even if they are not traditionally considered non-associative algebraists. Since its inception in 2011, NAART has been committed to fostering cross-disciplinary connections in the study of non-associative structures.

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c1arification has brought the proof of the gems in this subject to a level that can be appreciated by students with only modest background. The subject is almost unique in the wide range of contacts that it makes with other parts of mathematics. The study of associative algebras con tributes to and draws from such topics as group theory, commutative ring theory, field theory, algebraic number theory, algebraic geometry, homo logical algebra, and category theory. It even has some ties with parts of applied mathematics.

non associative algebra: *Algebra and Applications 1* Abdenacer Makhlouf, 2021-03-12 This book is part of Algebra and Geometry, a subject within the SCIENCES collection published by ISTE and Wiley, and the first of three volumes specifically focusing on algebra and its applications. Algebra and Applications 1 centers on non-associative algebras and includes an introduction to derived categories. The chapters are written by recognized experts in the field, providing insight into new trends, as well as a comprehensive introduction to the theory. The book incorporates self-contained surveys with the main results, applications and perspectives. The chapters in this volume cover a wide variety of algebraic structures and their related topics. Jordan superalgebras, Lie algebras, composition algebras, graded division algebras, non-associative C*- algebras, H*-algebras, Krichever-Novikov type algebras, preLie algebras and related structures, geometric structures on 3-Lie algebras and derived categories are all explored. Algebra and Applications 1 is of great interest to graduate students and researchers. Each chapter combines some of the features of both a graduate level textbook and of research level surveys.

non associative algebra: Non-commutative and Non-associative Algebra and Analysis Structures Sergei Silvestrov, Anatoliy Malyarenko, 2023-09-25 The goal of the 2019 conference on Stochastic Processes and Algebraic Structures held in SPAS2019, Västerås, Sweden, from September 30th to October 2nd 2019 was to showcase the frontiers of research in several important topics of mathematics, mathematical statistics, and its applications. The conference has been organized along the following tracks: 1. Stochastic processes and modern statistical methods in theory and practice, 2. Engineering Mathematics, 3. Algebraic Structures and applications. This book highlights the latest advances in algebraic structures and applications focused on mathematical notions, methods, structures, concepts, problems, algorithms, and computational methods for the natural sciences, engineering, and modern technology. In particular, the book features mathematical methods and models from non-commutative and non-associative algebras and rings associated to generalizations of differential calculus, quantum deformations of algebras, Lie algebras, Lie superalgebras, color Lie algebras, Hom-algebras and their n-ary generalizations, semi-groups and group algebras, non-commutative and non-associative algebras and computational algebra interplay with q-special functions and q-analysis, topology, dynamical systems, representation theory, operator theory and functional analysis, applications of algebraic structures in coding theory, information analysis, geometry and probability theory. The book gathers selected, high-quality contributed chapters from several large research communities working on modern algebraic structures and their applications. The chapters cover both theory and applications, and are illustrated with a wealth of ideas, theorems, notions, proofs, examples, open problems, and results on the interplay of algebraic structures with other parts of Mathematics. The applications help readers grasp the material, and encourage them to develop new mathematical methods and concepts in their future research. Presenting new methods and results, reviews of cutting-edge research, open problems, and directions for future research, will serve as a source of inspiration for a broad range of researchers and students.

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aspects interplay in the same cadence, promoting interactions between theory and applications, and between nonassociative algebraic structures and various fields in pure and applied mathematics. In this volume, the reader will find novel studies on topics such as left almost algebras, logical algebras, groupoids and their generalizations, algebraic geometry and its relations with quiver algebras, enumerative combinatorics, representation theory, fuzzy logic and foundation theory, fuzzy algebraic structures, group amalgams, computer-aided development and transformation of the theory of nonassociative algebraic structures, and applications within natural sciences and engineering. Researchers and graduate students in algebraic structures and their applications can hugely benefit from this book, which can also interest any researcher exploring multi-disciplinarity and complexity in the scientific realm.

non associative algebra: New Perspectives in Algebra, Topology and Categories Maria Manuel Clementino, Alberto Facchini, Marino Gran, 2021-10-16 This book provides an introduction to some key subjects in algebra and topology. It consists of comprehensive texts of some hours courses on the preliminaries for several advanced theories in (categorical) algebra and topology. Often, this kind of presentations is not so easy to find in the literature, where one begins articles by assuming a lot of knowledge in the field. This volume can both help young researchers to quickly get into the subject by offering a kind of « roadmap » and also help master students to be aware of the basics of other research directions in these fields before deciding to specialize in one of them. Furthermore, it can be used by established researchers who need a particular result for their own research and do not want to go through several research papers in order to understand a single proof. Although the chapters can be read as « self-contained » chapters, the authors have tried to coordinate the texts in order to make them complementary. The seven chapters of this volume correspond to the seven courses taught in two Summer Schools that took place in Louvain-la-Neuve in the frame of the project Fonds d'Appui à l'Internationalisation of the Université catholique de Louvain to strengthen the collaborations with the universities of Coimbra, Padova and Poitiers, within the Coimbra Group.

non associative algebra: *Relation Algebras by Games* Robin Hirsch, Ian Hodkinson, 2002-08-15 In part 2, games are introduced, and used to axiomatise various classes of algebras. Part 3 discusses approximations to representability, using bases, relation algebra reducts, and relativised representations. Part 4 presents some constructions of relation algebras, including Monk algebras and the 'rainbow construction', and uses them to show that various classes of representable algebras are non-finitely axiomatisable or even non-elementary. Part 5 shows that the representability problem for finite relation algebras is undecidable, and then in contrast proves some finite base property results. Part 6 contains a condensed summary of the book, and a list of problems. There are more than 400 exercises. P The book is generally self-contained on relation algebras and on games, and introductory text is scattered throughout. Some familiarity with elementary aspects of first-order logic and set theory is assumed, though many of the definitions are given.-

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Algebras Charles W. Curtis, Irving Reiner, 2006 Provides an introduction to various aspects of the representation theory of finite groups. This book covers such topics as general non-commutative algebras, Frobenius algebras, representations over non-algebraically closed fields and fields of non-zero characteristic, and integral representations.

non associative algebra: A Taste of Jordan Algebras Kevin McCrimmon, 2006-05-29 This book describes the history of Jordan algebras and describes in full mathematical detail the recent structure theory for Jordan algebras of arbitrary dimension due to Efim Zel'manov. Jordan algebras crop up in many surprising settings, and find application to a variety of mathematical areas. No knowledge is required beyond standard first-year graduate algebra courses.

non associative algebra: Algebraic Structures and Applications Sergei Silvestrov, Anatoliy Malyarenko, Milica Rančić, 2020-06-18 This book explores the latest advances in algebraic structures and applications, and focuses on mathematical concepts, methods, structures, problems, algorithms and computational methods important in the natural sciences, engineering and modern technologies. In particular, it features mathematical methods and models of non-commutative and non-associative algebras, hom-algebra structures, generalizations of differential calculus, quantum deformations of algebras, Lie algebras and their generalizations, semi-groups and groups, constructive algebra, matrix analysis and its interplay with topology, knot theory, dynamical systems, functional analysis, stochastic processes, perturbation analysis of Markov chains, and applications in network analysis, financial mathematics and engineering mathematics. The book addresses both theory and applications, which are illustrated with a wealth of ideas, proofs and examples to help readers understand the material and develop new mathematical methods and concepts of their own. The high-quality chapters share a wealth of new methods and results, review cutting-edge research and discuss open problems and directions for future research. Taken together, they offer a source of inspiration for a broad range of researchers and research students whose work involves algebraic structures and their applications, probability theory and mathematical statistics, applied mathematics, engineering mathematics and related areas.

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emphasizes algebras that are, for the most part, finite-dimensional over a field. We are delighted to publish this classic book as part of our extensive Classic Library collection. Many of the books in our collection have been out of print for decades, and therefore have not been accessible to the general public. The aim of our publishing program is to facilitate rapid access to this vast reservoir of literature, and our view is that this is a significant literary work, which deserves to be brought back into print after many decades. The contents of the vast majority of titles in the Classic Library have been scanned from the original works. To ensure a high quality product, each title has been meticulously hand curated by our staff. Our philosophy has been guided by a desire to provide the reader with a book that is as close as possible to ownership of the original work. We hope that you will enjoy this wonderful classic work, and that for you it becomes an enriching experience.

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orientations and C*-products, referenced as [AS] in the sequel, gives an account of the necessary prerequisites on C*-algebras and von Neumann algebras, as well as a discussion of the key notion of orientations of state spaces. For the convenience of the reader, we have summarized these prerequisites in an appendix which contains all relevant definitions and results (listed as (AI), (A2), ...), with reference back to [AS] for proofs, so that this book is self-contained.

non associative algebra: Non-associative Structures and Other Related Structures Florin Felix Nichita, 2020-06-16 Leonhard Euler (1707–1783) was born in Basel, Switzerland. Euler's formula is a mathematical formula in complex analysis that establishes the fundamental relationship between the trigonometric functions and the complex exponential function. When its variable is the number pi, Euler's formula evaluates to Euler's identity. On the other hand, the Yang–Baxter equation is considered the most beautiful equation by many scholars. In this book, we study connections between Euler's formulas and the Yang–Baxter equation. Other interesting sections include: non-associative algebras with metagroup relations; branching functions for admissible representations of affine Lie Algebras; super-Virasoro algebras; dual numbers; UJLA structures; etc.

non associative algebra: *Evolution Algebras and Their Applications* Jianjun Paul Tian, 2008 Behind genetics and Markov chains, there is an intrinsic algebraic structure. It is defined as a type of new algebra: as evolution algebra. This concept lies between algebras and dynamical systems. Algebraically, evolution algebras are non-associative Banach algebras; dynamically, they represent discrete dynamical systems. Evolution algebras have many connections with other mathematical fields including graph theory, group theory, stochastic processes, dynamical systems, knot theory, 3-manifolds, and the study of the Ihara-Selberg zeta function. In this volume the foundation of evolution algebra theory and applications in non-Mendelian genetics and Markov chains is developed, with pointers to some further research topics.

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non associative algebra: *Structure of Algebras* Abraham Adrian Albert, 1939-12-31 The first three chapters of this work contain an exposition of the Wedderburn structure theorems. Chapter IV contains the theory of the commutator subalgebra of a simple subalgebra of a normal simple algebra, the study of automorphisms of a simple algebra, splitting fields, and the index reduction factor theory. The fifth chapter contains the foundation of the theory of crossed products and of their special case, cyclic algebras. The theory of exponents is derived there as well as the consequent factorization of normal division algebras into direct factors of prime-power degree. Chapter VI consists of the study of the abelian group of cyclic systems which is applied in Chapter VII to yield the theory of the structure of direct products of cyclic algebras and the consequent properties of norms in cyclic fields. This chapter is closed with the theory of p-algebras. In Chapter VIII an exposition is given of the theory of the representations of algebras. The treatment is somewhat novel in that while the recent expositions have used representation theorems to obtain a number of results

on algebras, here the theorems on algebras are themselves used in the derivation of results on representations. The presentation has its inspiration in the author's work on the theory of Riemann matrices and is concluded by the introduction to the generalization (by H. Weyl and the author) of that theory. The theory of involutorial simple algebras is derived in Chapter X both for algebras over general fields and over the rational field. The results are also applied in the determination of the structure of the multiplication algebras of all generalized Riemann matrices, a result which is seen in Chapter XI to imply a complete solution of the principal problem on Riemann matrices.

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text for a graduate course in commutative algebra, focusing on the algebraic underpinnings of algebraic geometry and of number theory. Accordingly, the theory of affine algebras is featured, treated both directly and via the theory of Noetherian and Artinian modules, and the theory of graded algebras is included to provide the foundation for projective varieties. Major topics include the theory of modules over a principal ideal domain, and its applicationsto matrix theory (including the Jordan decomposition), the Galois theory of field extensions, transcendence degree, the prime spectrum of an algebra, localization, and the classical theory of Noetherian and Artinian rings. Later chapters include some algebraic theory of elliptic curves (featuring theMordell-Weil theorem) and valuation theory, including local fields. One feature of the book is an extension of the text through a series of appendices. This permits the inclusion of more advanced material, such as transcendental field extensions, the discriminant and resultant, the theory of Dedekind domains, and basic theorems of rings of algebraic integers. An extended appendix on derivations includes the Jacobian conjecture and Makar-Limanov's theory of locally nilpotent derivations. Grobnerbases can be found in another appendix. Exercises provide a further extension of the text. The book can be used both as a textbook and as a reference source.

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theory of groups with analysis. The idea of applying analytical methods to the theory of polynomial identities appeared in the early 1970s and this approach has become one of the most powerful tools of the theory. A PI-algebra is any algebra satisfying at least one nontrivial polynomial identity. This includes the polynomial rings in one or several variables, the Grassmann algebra, finite-dimensional algebras, and many other algebras occurring naturally in mathematics. The core of the book is the proof that the sequence of co-dimensions of any PI-algebra has integral exponential growth - the PI-exponent of the algebra. Later chapters further apply these results to subjects such as a characterization of varieties of algebras having polynomial growth and a classification of varieties that are minimal for a given exponent.

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