# module algebra

## Delving into the Intriguing World of Module Algebra

Are you intrigued by the elegant interplay of abstract algebra and linear algebra? Do you find yourself captivated by the power of structures that seemingly defy intuition? Then prepare to embark on a journey into the fascinating world of module algebra. This comprehensive guide will unravel the mysteries of modules, exploring their fundamental concepts, key properties, and practical applications. We'll navigate through definitions, theorems, and examples, equipping you with a solid understanding of this vital area of mathematics. Get ready to unlock the secrets of module algebra!

### What is Module Algebra?

At its core, module algebra explores the algebraic structures known as modules. A module is a generalization of the familiar concept of a vector space, but instead of being defined over a field (like real or complex numbers), it's defined over a ring. This seemingly small change opens up a vast landscape of possibilities and allows us to study algebraic structures with richer and more complex properties. Think of it as extending the familiar tools of linear algebra to a wider, more nuanced algebraic context.

A module M over a ring R (denoted as R-module) is an abelian group (M, +) equipped with an operation of scalar multiplication: $R \times M \to M$, satisfying the following axioms:

Distributivity: For all $r, s \in R$ and $x, y \in M$: $r(x + y) = rx + ry$ and $(r + s)x = rx + sx$.
Associativity: For all $r, s \in R$ and $x \in M$: $(rs)x = r(sx)$.
Scalar multiplication identity: For all $x \in M$: $1x = x$, where 1 is the multiplicative identity in the ring R.

### Types of Modules: A Deeper Dive

Modules exhibit a rich diversity of structures, and classifying them is a significant area of study. Some key types include:

Free Modules: These modules are analogous to vector spaces with a basis. Every element can be uniquely expressed as a linear combination of a set of linearly independent elements (the basis). Free modules are fundamental building blocks in the study of modules.

Cyclic Modules: These are modules generated by a single element. Every element can be expressed as a multiple of this generating element. Understanding cyclic modules provides crucial insight into the structure of more complex modules.

Simple Modules: These are non-zero modules with no non-trivial submodules. They are the "atoms" of module theory, analogous to simple groups in group theory. Their analysis is crucial for understanding the structure of more general modules.

Noetherian and Artinian Modules: These modules possess crucial finiteness conditions related to chains of submodules. Noetherian modules have ascending chain conditions, while Artinian modules have descending chain conditions. These properties are essential for developing powerful decomposition theorems.

## Key Concepts and Theorems

Module algebra is not simply about definitions; it's a rich tapestry woven with powerful theorems and concepts. Some key elements include:

Submodules: A submodule is a subgroup of a module that is also closed under scalar multiplication. Submodules play a role analogous to subspaces in linear algebra.

Homomorphisms and Isomorphisms: Module homomorphisms are structure-preserving maps between modules, while isomorphisms are bijective homomorphisms. They are crucial for comparing and classifying modules.

Direct Sums and Direct Products: These constructions allow us to build larger modules from smaller ones, leading to a deeper understanding of module structure.

Module Decomposition Theorems: These theorems, like the Krull-Schmidt theorem, provide ways to decompose modules into simpler, indecomposable modules, offering crucial insights into their internal structure.

## Applications of Module Algebra

Module algebra isn't merely an abstract mathematical construct; it has significant applications in various fields:

Representation Theory: Module theory forms the foundation of representation theory, a vital tool in group theory, allowing us to study groups by representing them as groups of linear transformations on vector spaces (modules).

Algebraic Geometry: Modules play a critical role in algebraic geometry, providing tools for studying algebraic varieties and their properties.

Ring Theory: The study of rings is intrinsically linked to the study of modules, as modules provide a rich context for understanding the structure of rings.

Computer Science: Concepts from module theory find applications in areas like coding theory and cryptography.

## A Textbook Example: "Introduction to Module Theory"

Contents:

Introduction: Brief overview of rings, modules, and motivations for studying module theory.
Chapter 1: Basic Definitions and Properties: Detailed exploration of module axioms, examples, and fundamental properties.
Chapter 2: Submodules and Quotient Modules: Deep dive into submodule structure, including quotient

modules and isomorphism theorems.
Chapter 3: Homomorphisms and Isomorphisms: Detailed exploration of module homomorphisms, isomorphisms, and their properties.
Chapter 4: Direct Sums and Products: Construction of larger modules from smaller ones, exploring direct sum and product decompositions.
Chapter 5: Free Modules and Bases: Exploration of free modules, bases, and rank, including connections to linear algebra.
Chapter 6: Noetherian and Artinian Modules: Analysis of chain conditions and their implications for module structure.
Chapter 7: Simple Modules and Semisimple Modules: Study of simple modules as building blocks and decomposition into semisimple modules.
Chapter 8: Applications and Advanced Topics: Introduction to applications in representation theory, algebraic geometry, and other areas.
Conclusion: Summary of key concepts and future directions in module theory.

## Detailed Explanation of Textbook Chapters (Excerpt)

Let's delve into a few key chapters to illustrate the depth and breadth of module theory.

Chapter 1: Basic Definitions and Properties: This chapter lays the groundwork. We rigorously define modules over rings, explore various examples (like modules over fields, which are just vector spaces!), and prove elementary but crucial properties, such as the uniqueness of the zero element in a module.

Chapter 3: Homomorphisms and Isomorphisms: Here we introduce module homomorphisms – functions that preserve the module structure. Isomorphisms are bijective homomorphisms, essentially showing that two modules are structurally identical. We prove fundamental isomorphism theorems, analogous to those in group theory, which are crucial for classifying modules.

Chapter 5: Free Modules and Bases: This chapter explores free modules – modules with a basis, much like vector spaces. Understanding free modules is paramount because many modules can be understood in terms of their relationship to free modules. We delve into the concept of rank and its properties.

Chapter 7: Simple Modules and Semisimple Modules: Simple modules are the "atoms" of module theory – they have no non-trivial submodules. This chapter explores their properties and introduces the crucial concept of semisimple modules, which can be decomposed into direct sums of simple modules. This decomposition is a cornerstone of module theory, allowing us to understand the structure of complex modules by breaking them down into their simplest constituents.

## FAQs

1. What's the difference between a module and a vector space? A vector space is a module over a field; a module is a generalization to rings. Rings can lack the multiplicative inverse property that fields possess.

1. Why are modules important in algebra? Modules provide a powerful framework for understanding the structure of rings and groups, leading to crucial results in representation theory and other areas.

1. Are there different types of modules? Yes, many types exist, including free modules, cyclic modules, simple modules, Noetherian modules, and Artinian modules, each with unique properties.

1. What are submodules? Submodules are subsets of a module that are themselves modules under the inherited operations.

1. How are modules used in representation theory? Groups are represented as groups of linear transformations on modules, providing a powerful method to study group structure.

1. What are homomorphisms in the context of modules? They are structure-preserving maps between modules, crucial for comparing and classifying modules.

1. What are direct sums and products of modules? They are ways to construct larger modules from smaller ones, vital for understanding module decomposition.

1. What are some applications of module theory outside pure mathematics? It finds applications in coding theory, cryptography, and computer algebra systems.

1. Are there any specific theorems that are essential to understanding module theory? The Krull-Schmidt theorem and the various isomorphism theorems are foundational.

## Related Articles:

1. Ring Theory Fundamentals: A comprehensive introduction to ring theory, providing the

necessary background for understanding modules.

1. Vector Spaces: A Refresher: A review of vector space concepts to build a solid foundation before delving into modules.

1. Group Theory and its Applications: Exploring the connections between group theory and module theory, particularly in representation theory.

1. Introduction to Representation Theory: A detailed exploration of representation theory, highlighting the crucial role of modules.

1. Noetherian Rings and Modules: A focused study of Noetherian rings and their corresponding modules.

1. Artinian Rings and Modules: A focused study of Artinian rings and their corresponding modules.

1. Module Decomposition Theorems: A detailed examination of various decomposition theorems, including the Krull-Schmidt theorem.

1. Applications of Module Theory in Cryptography: Exploring specific uses of module theory in cryptographic systems.

1. Advanced Topics in Module Theory: A glimpse into more advanced concepts and research areas within module algebra.

**module algebra:** *Algebra* William A. Adkins, Steven H. Weintraub, 1992 First year graduate algebra text. The choice of topics is guided by the underlying theme of modules as a basic unifying concept in mathematics. Beginning with standard topics in group and ring theory, the authors then develop basic module theory and its use in investigating bilinear, sesquilinear, and quadratic forms. Annotation copyrighted by Book News, Inc., Portland, OR

**module algebra: Module Theory** Thomas Scott Blyth, 1990 This textbook provides a self-contained course on the basic properties of modules and their importance in the theory of linear algebra. The first 11 chapters introduce the central results and applications of the theory of modules. Subsequent chapters deal with advanced linear algebra, including multilinear and tensor algebra, and explore such topics as the exterior product approach to the determinants of matrices, a module-theoretic approach to the structure of finitely generated Abelian groups, canonical forms, and normal transformations. Suitable for undergraduate courses, the text now includes a proof of the celebrated Wedderburn-Artin theorem which determines the structure of simple Artinian rings.

**module algebra:** *Algebra* William A. Adkins, Steven H. Weintraub, 2012-12-06 This book is designed as a text for a first-year graduate algebra course. As necessary background we would consider a good undergraduate linear algebra course. An undergraduate abstract algebra course, while helpful, is not necessary (and so an adventurous undergraduate might learn some algebra from this book). Perhaps the principal distinguishing feature of this book is its point of view. Many textbooks tend to be encyclopedic. We have tried to write one that is thematic, with a consistent point of view. The theme, as indicated by our title, is that of modules (though our intention has not been to write a textbook purely on module theory). We begin with some group and ring theory, to set the stage, and then, in the heart of the book, develop module theory. Having developed it, we present some of its applications: canonical forms for linear transformations, bilinear forms, and group representations. Why modules? The answer is that they are a basic unifying concept in mathematics. The reader is probably already familiar with the basic role that vector spaces play in mathematics, and modules are a generaliza tion of vector spaces. (To be precise, modules are to rings as vector spaces are to fields.

**module algebra:** *Modules and Rings* John Dauns, 1994-10-28 This book on modern module and non-commutative ring theory is ideal for beginning graduate students. It starts at the foundations of the subject and progresses rapidly through the basic concepts to help the reader reach current research frontiers. Students will have the chance to develop proofs, solve problems, and to find interesting questions. The first half of the book is concerned with free, projective, and injective modules, tensor algebras, simple modules and primitive rings, the Jacobson radical, and subdirect products. Later in the book, more advanced topics, such as hereditary rings, categories and functors, flat modules, and purity are introduced. These later chapters will also prove a useful reference for researchers in non-commutative ring theory. Enough background material (including detailed proofs) is supplied to give the student a firm grounding in the subject.

**module algebra:** *Foundations of Module and Ring Theory* Robert Wisbauer, 2018-05-11 This volume provides a comprehensive introduction to module theory and the related part of ring theory, including original results as well as the most recent work. It is a useful and stimulating study for those new to the subject as well as for researchers and serves as a reference volume. Starting form a basic understanding of linear algebra, the theory is presented and accompanied by complete proofs. For a module M, the smallest Grothendieck category containing it is denoted by o[M] and module theory is developed in this category. Developing the techniques in o[M] is no more complicated than in full module categories and the higher generality yields significant advantages: for example, module theory may be developed for rings without units and also for non-associative rings. Numerous exercises are included in this volume to give further insight into the topics covered and to draw attention to related results in the literature.

**module algebra:** Modules and Algebras Robert Wisbauer, 1996-05-15 Module theory over commutative asociative rings is usually extended to noncommutative associative rings by introducing the category of left (or right) modules. An alternative to this procedure is suggested by considering bimodules. A refined module theory for associative rings is used to investigate the bimodule structure of arbitary algebras and group actions on these algebras.

**module algebra:** *Unstable Modules Over the Steenrod Algebra and Sullivan's Fixed Point Set Conjecture* Lionel Schwartz, 1994-07-15 A comprehensive account of one of the main directions of algebraic topology, this book focuses on the Sullivan conjecture and its generalizations and applications. Lionel Schwartz collects here for the first time some of the most innovative work on the theory of modules over the Steenrod algebra, including ideas on the Segal conjecture, work from the late 1970s by Adams and Wilkerson, and topics in algebraic group representation theory. This course-tested book provides a valuable reference for algebraic topologists and includes foundational material essential for graduate study.

**module algebra: Ring and Module Theory** Toma Albu, Gary F. Birkenmeier, Ali Erdogan, Adnan Tercan, 2011-02-04 This book is a collection of invited papers and articles, many presented at the 2008 International Conference on Ring and Module Theory. The papers explore the latest in various areas of algebra, including ring theory, module theory and commutative algebra.

**module algebra: Algebras, Rings and Modules** Michiel Hazewinkel, Nadezhda Mikhaĭlovna Gubareni, Vladimir V. Kirichenko, 2010 Presenting an introduction to the theory of Hopf algebras, the authors also discuss some important aspects of the theory of Lie algebras. This book includes a chapters on the Hopf algebra of symmetric functions, the Hopf algebra of representations of the symmetric groups, the Hopf algebras of the nonsymmetric and quasisymmetric functions, and the Hopf algebra of permutations.

**module algebra: A Primer of Algebraic D-Modules** S. C. Coutinho, 1995-09-07 The theory of D-modules is a rich area of study combining ideas from algebra and differential equations, and it has significant applications to diverse areas such as singularity theory and representation theory. This book introduces D-modules and their applications avoiding all unnecessary over-sophistication. It is aimed at beginning graduate students and the approach taken is algebraic, concentrating on the role of the Weyl algebra. Very few prerequisites are assumed, and the book is virtually self-contained. Exercises are included at the end of each chapter and the reader is given ample references to the more advanced literature. This is an excellent introduction to D-modules for all who are new to this area.

**module algebra: Semidistributive Modules and Rings** Askar A. Tuganbaev, 1998 Introduces structural and homological methods of ring theory, and explains the relationship between semidistributive modules and flat, projective, injective, multiplication, and Bezout modules. Contains chapters on areas such as radicals and semisimple modules, rings of quotients, flat modules and semiperfect rings, semiheridity and invariant rings, endomorphism rings, skew-injective rings, and monoid rings. Includes chapter exercises. Background to the material can be found in most graduate level texts in algebra. Annotation copyrighted by Book News, Inc., Portland, OR

**module algebra:** Extensions of Rings and Modules Gary F. Birkenmeier, Jae Keol Park, S Tariq Rizvi, 2013-07-19 The extensions of rings and modules have yet to be explored in detail in a research monograph. This book presents state of the art research and also stimulating new and further research. Broken into three parts, Part I begins with basic notions, terminology, definitions and a description of the classes of rings and modules. Part II considers the transference of conditions between a base ring or module and its extensions. And Part III utilizes the concept of a minimal essental extension with respect to a specific class (a hull). Mathematical interdisciplinary applications appear throughout. Major applications of the ring and module theory to Functional Analysis, especially C*-algebras, appear in Part III, make this book of interest to Algebra and Functional Analysis researchers. Notes and exercises at the end of every chapter, and open problems at the end of all three parts, lend this as an ideal textbook for graduate or advanced undergradate students.

**module algebra:** A First Course In Module Theory Mike E Keating, 1998-07-31 This book is an introduction to module theory for the reader who knows something about linear algebra and ring theory. Its main aim is the derivation of the structure theory of modules over Euclidean domains. This theory is applied to obtain the structure of abelian groups and the rational canonical and Jordan normal forms of matrices. The basic facts about rings and modules are given in full generality, so that some further topics can be discussed, including projective modules and the connection between modules and representations of groups.The book is intended to serve as supplementary reading for the third or fourth year undergraduate who is taking a course in module theory. The further topics point the way to some projects that might be attempted in conjunction with a taught course.

**module algebra: Approximations and Endomorphism Algebras of Modules** Rüdiger Göbel, Jan Trlifaj, 2012-10-01 This second, revised and substantially extended edition of Approximations and Endomorphism Algebras of Modules reflects both the depth and the width of recent developments in the area since the first edition appeared in 2006. The new division of the monograph into two volumes roughly corresponds to its two central topics, approximation theory (Volume 1) and realization theorems for modules (Volume 2). It is a widely accepted fact that the category of all modules over a general associative ring is too complex to admit classification. Unless the ring is of finite representation type we must limit attempts at classification to some restricted subcategories of modules. The wild character of the category of all modules, or of one of its subcategories C, is often indicated by the presence of a realization theorem, that is, by the fact that any reasonable algebra is isomorphic to the endomorphism algebra of a module from C. This results in the existence of pathological direct sum decompositions, and these are generally viewed as obstacles to classification. In order to overcome this problem, the approximation theory of modules has been developed. The idea here is to select suitable subcategories C whose modules can be classified, and then to approximate arbitrary modules by those from C. These approximations are neither unique nor functorial in general, but there is a rich supply available appropriate to the requirements of various particular applications. The authors bring the two theories together. The first volume, Approximations, sets the scene in Part I by introducing the main classes of modules relevant here: the S-complete, pure-injective, Mittag-Leffler, and slender modules. Parts II and III of the first volume develop the key methods of approximation theory. Some of the recent applications to the structure of modules are also presented here, notably for tilting, cotilting, Baer, and Mittag-Leffler modules. In the second volume, Predictions, further basic instruments are introduced: the prediction principles, and their applications to proving realization theorems. Moreover, tools are developed there for answering problems motivated in algebraic topology. The authors concentrate on the impossibility of classification for modules over general rings. The wild character of many categories C of modules is documented here by the realization theorems that represent critical R-algebras over commutative rings R as endomorphism algebras of modules from C. The monograph starts from basic facts and gradually develops the theory towards its present frontiers. It is suitable both for graduate students interested in algebra and for experts in module and representation theory.

**module algebra: Module Theory, Extending Modules and Generalizations** Adnan Tercan, Canan C. Yücel, 2016-05-13 The main focus of this monograph is to offer a comprehensive presentation of known and new results on various generalizations of CS-modules and CS-rings. Extending (or CS) modules are generalizations of injective (and also semisimple or uniform) modules. While the theory of CS-modules is well documented in monographs and textbooks, results on generalized forms of the CS property as well as dual notions are far less present in the literature. With their work the authors provide a solid background to module theory, accessible to anyone familiar with basic abstract algebra. The focus of the book is on direct sums of CS-modules and classes of modules related to CS-modules, such as relative (injective) ejective modules, (quasi) continuous modules, and lifting modules. In particular, matrix CS-rings are studied and clear proofs of fundamental decomposition results on CS-modules over commutative domains are given, thus complementing existing monographs in this area. Open problems round out the work and establish the basis for further developments in the field. The main text is complemented by a wealth of examples and exercises.

**module algebra: Modules and the Structure of Rings** Golan, 1991-04-24 This book offers vital background information on methods for solving hard classification problems of algebraic structures. It explains how algebraists deal with the problem of the structure of modules over rings and how they make use of these structures to classify rings.

**module algebra:** *Modules and the Structure of Rings* Golan, 2017-10-19 This textbook is designed for students with at least one solid semester of abstract algebra,some linear algebra background, and no previous knowledge of module theory. Modulesand the Structure of Rings details the use of modules over a ring as a means of consideringthe structure of the ring itself--explaining the mathematics and inductivereasoning used in working on ring theory challenges and emphasizing modules insteadof rings.Stressing the inductive aspect of mathematical research underlying the formal deductivestyle of the literature, this volume offers vital background on current methods for solvinghard classification problems of algebraic structures. Written in an informal butcompletely rigorous style, Modules and the Structure of Rings clarifies sophisticatedproofs ... avoids the formalism of category theory ... aids independent study or seminarwork ... and supplies end-of-chapter problems.This book serves as an excellent primary.text for upper-level undergraduate and graduatestudents in one-semester courses on ring or module theory-laying a foundation formore advanced study of homological algebra or module theory.

**module algebra:** Algebras and Modules I Idun Reiten, Sverre O. Smalø, Øyvind Solberg, Canadian Mathematical Society, 1998 Surveys developments in the representation theory of finite dimensional algebras and related topics in seven papers illustrating different techniques developed over the recent years. For graduate students and researchers with a background in commutative algebra, including rings, modules, and homological algebra. Suitable as a text for an advanced graduate course. No index. Member prices are $31 for institutions and $23 for individuals, and are available to members of the Canadian Mathematical Society. Annotation copyrighted by Book News, Inc., Portland, OR

**module algebra:** Model Theoretic Algebra With Particular Emphasis on Fields, Rings, Modules Christian.U Jensen, 2022-03-10 This volume highlights the links between model theory and algebra. The work contains a definitive account of algebraically compact modules, a topic of central importance for both module and model theory. Using concrete examples, particular emphasis is given to model theoretic concepts, such as axiomizability. Pure mathematicians, especially algebraists, ring theorists, logicians, model theorists and representation theorists, should find this an absorbing and stimulating book.

**module algebra: Rings and Their Modules** Paul E. Bland, 2011 This book is an introduction to the theory of rings and modules that goes beyond what one normally obtains in a graduate course in abstract algebra. In addition to the presentation of standard topics in ring and module theory, it also covers category theory, homological algebra and even more specialized topics like injective envelopes and proj

**module algebra:** Commutative Algebra: Constructive Methods Henri Lombardi, Claude Quitté, 2015-07-22 Translated from the popular French edition, this book offers a detailed introduction to various basic concepts, methods, principles, and results of commutative algebra. It takes a constructive viewpoint in commutative algebra and studies algorithmic approaches alongside several abstract classical theories. Indeed, it revisits these traditional topics with a new and simplifying manner, making the subject both accessible and innovative. The algorithmic aspects of such naturally abstract topics as Galois theory, Dedekind rings, Prüfer rings, finitely generated projective modules, dimension theory of commutative rings, and others in the current treatise, are all analysed in the spirit of the great developers of constructive algebra in the nineteenth century. This updated and revised edition contains over 350 well-arranged exercises, together with their helpful hints for solution. A basic knowledge of linear algebra, group theory, elementary number theory as well as the fundamentals of ring and module theory is required. Commutative Algebra: Constructive Methods will be useful for graduate students, and also researchers, instructors and theoretical computer scientists.

**module algebra: Algebraic D-modules** Armand Borel, 1987 Presented here are recent developments in the algebraic theory of D-modules. The book contains an exposition of the basic notions and operations of D-modules, of special features of coherent, holonomic, and regular holonomic D-modules, and of the Riemann-Hilbert correspondence. The theory of Algebraic D-modules has found remarkable applications outside of analysis proper, in particular to infinite dimensional representations of semisimple Lie groups, to representations of Weyl groups, and to algebraic geometry.

**module algebra: Lectures on Rings and Modules** Joachim Lambek, 2009 This book is an introduction to the theory of associative rings and their modules, designed primarily for graduate students. The standard topics on the structure of rings are covered, with a particular emphasis on the concept of the complete ring of quotients. A survey of the fundamental concepts of algebras in the first chapter helps to make the treatment self-contained. The topics covered include selected results on Boolean and other commutative rings, the classical structure theory of associative rings, injective modules, and rings of quotients. The final chapter provides an introduction to homological algebra. Besides three appendices on further results, there is a six-page section of historical comments. Table of Contents: Fundamental Concepts of Algebra: 1.1 Rings and related algebraic systems; 1.2 Subrings, homomorphisms, ideals; 1.3 Modules, direct products, and direct sums; 1.4 Classical isomorphism theorems. Selected Topics on Commutative Rings: 2.1 Prime ideals in commutative rings; 2.2 Prime ideals in special commutative rings; 2.3 The complete ring of quotients of a commutative ring; 2.4 Rings of quotients of commutative semiprime rings; 2.5 Prime ideal spaces.Classical Theory of Associative Rings: 3.1 Primitive rings; 3.2 Radicals; 3.3 Completely reducible modules; 3.4 Completely reducible rings; 3.5 Artinian and Noetherian rings; 3.6 On lifting idempotents; 3.7 Local and semiperfect rings. Injectivity and Related Concepts: 4.1 Projective modules; 4.2 Injective modules; 4.3 The complete ring of quotients; 4.4 Rings of endomorphisms of injective modules; 4.5 Regular rings of quotients; 4.6 Classical rings of quotients; 4.7 The Faith-Utumi theorem. Introduction to Homological Algebra: 5.1 Tensor products of modules; 5.2 Hom and $\otimes$ as functors; 5.3 Exact sequences; 5.4 Flat modules; 5.5 Torsion and extension products. Appendixes; Comments; Bibliography; Index. Review from Zentralblatt Math: Due to their clarity and intelligible presentation, these lectures on rings and modules are a particularly successful introduction to the surrounding circle of ideas. Review from American Mathematical Monthly: An introduction to associative rings and modules which requires of the reader only the mathematical maturity which one would attain in a first-year graduate algebra [course]...in order to make the contents of the book as accessible as possible, the author develops all the fundamentals he will need.In addition to covering the basic topics...the author covers some topics not so readily available to the nonspecialist...the chapters are written to be as independent as possible...[which will be appreciated by] students making their first acquaintance with the subject...one of the most successful features of the book is that it can be read by graduate students with little or no help from a specialist. (CHEL/283.H)

**module algebra:** *Hopf Algebras* David E Radford, 2011-12-28 The book provides a detailed account of basic coalgebra and Hopf algebra theory with emphasis on Hopf algebras which are pointed, semisimple, quasitriangular, or are of certain other quantum groups. It is intended to be a graduate text as well as a research monograph.

**module algebra: Exercises in Modules and Rings** T.Y. Lam, 2009-12-08 This volume offers a compendium of exercises of varying degree of difficulty in the theory of modules and rings. It is the companion volume to GTM 189. All exercises are solved in full detail. Each section begins with an introduction giving the general background and the theoretical basis for the problems that follow.

**module algebra: Groups, Rings And Modules With Applications** M.R. Adhikari, A. Adhikari, 2003

**module algebra: Introduction To Commutative Algebra** Michael F. Atiyah, I.G. MacDonald, 2018-03-09 First Published in 2018. This book grew out of a course of lectures given to third year undergraduates at Oxford University and it has the modest aim of producing a rapid introduction to the subject. It is designed to be read by students who have had a first elementary course in general algebra. On the other hand, it is not intended as a substitute for the more voluminous tracts such as

Zariski-Samuel or Bourbaki. We have concentrated on certain central topics, and large areas, such as field theory, are not touched. In content we cover rather more ground than Northcott and our treatment is substantially different in that, following the modern trend, we put more emphasis on modules and localization.

**module algebra: Galois Module Structure of Algebraic Integers** Albrecht Fröhlich, 1983

**module algebra: Azumaya Algebras, Actions, and Modules** Darrell Haile, 1992 This volume contains the proceedings of a conference in honor of Goro Azumaya's seventieth birthday, held at Indiana University of Bloomington in May 1990. Professor Azumaya, who has been on the faculty of Indiana University since 1968, has made many important contributions to modern abstract algebra. His introduction and investigation of what have come to be known as Azumaya algebras subsequently stimulated much research on such rings and algebras, as well as applications to geometry and number theory. In addition to honoring Professor Azumaya's contributions, the conference was intended to stimulate interaction among three areas of his research interests; Azumaya algebras, group and Hopf algebra actions, and module theory. Aimed at researchers in algebra, this volume contains contributions by some of the leaders in these areas.

**module algebra: Algebra** Thomas W. Hungerford, 2003-02-14 Finally a self-contained, one volume, graduate-level algebra text that is readable by the average graduate student and flexible enough to accommodate a wide variety of instructors and course contents. The guiding principle throughout is that the material should be presented as general as possible, consistent with good pedagogy. Therefore it stresses clarity rather than brevity and contains an extraordinarily large number of illustrative exercises.

**module algebra: Linear Algebra over Commutative Rings** Bernard R. McDonald, 2020-11-26 This monograph arose from lectures at the University of Oklahoma on topics related to linear algebra over commutative rings. It provides an introduction of matrix theory over commutative rings. The monograph discusses the structure theory of a projective module.

**module algebra: Algebras, Rings and Modules** Michiel Hazewinkel, Nadiya M. Gubareni, 2016-04-05 The theory of algebras, rings, and modules is one of the fundamental domains of modern mathematics. General algebra, more specifically non-commutative algebra, is poised for major advances in the twenty-first century (together with and in interaction with combinatorics), just as topology, analysis, and probability experienced in the twentieth centu

**module algebra:** Lectures on Modules and Rings Tsit-Yuen Lam, 2012-12-06 This new book can be read independently from the first volume and may be used for lecturing, seminar- and self-study, or for general reference. It focuses more on specific topics in order to introduce readers to a wealth of basic and useful ideas without the hindrance of heavy machinery or undue abstractions. User-friendly with its abundance of examples illustrating the theory at virtually every step, the volume contains a large number of carefully chosen exercises to provide newcomers with practice, while offering a rich additional source of information to experts. A direct approach is used in order to present the material in an efficient and economic way, thereby introducing readers to a considerable amount of interesting ring theory without being dragged through endless preparatory material.

**module algebra:** Hopf Algebras Jeffrey Bergen, Stefan Catoiu, William Chin, 2004-01-28 This volume publishes key proceedings from the recent International Conference on Hopf Algebras held at DePaul University, Chicago, Illinois. With contributions from leading researchers in the field, this collection deals with current topics ranging from categories of infinitesimal Hopf modules and bimodules to the construction of a Hopf algebraic Morita invariant. It uses the newly introduced theory of bi-Frobenius algebras to investigate a notion of group-like algebras and summarizes results on the classification of Hopf algebras of dimension pq. It also explores pre-Lie, dendriform, and Nichols algebras and discusses support cones for infinitesimal group schemes.

**module algebra:** *Yetter-Drinfel'd Hopf Algebras over Groups of Prime Order* Yorck Sommerhäuser, 2004-10-19 Being the first monograph devoted to this subject, the book addresses the classification problem for semisimple Hopf algebras, a field that has attracted considerable attention in the last years. The special approach to this problem taken here is via semidirect product decompositions into Yetter-Drinfel'd Hopf algebras and group rings of cyclic groups of prime order. One of the main features of the book is a complete treatment of the structure theory for such Yetter-Drinfel'd Hopf algebras.

**module algebra: The Algebraic Theory of Modular Systems** Francis Sowerby Macaulay, 1916

**module algebra:** *Handbook of Algebra* M. Hazewinkel, 2009-07-08 Algebra, as we know it today, consists of many different ideas, concepts and results. A reasonable estimate of the number of these different items would be somewhere between 50,000 and 200,000. Many of these have been named and many more could (and perhaps should) have a name or a convenient designation. Even the nonspecialist is likely to encounter most of these, either somewhere in the literature, disguised as a definition or a theorem or to hear about them and feel the need for more information. If this happens, one should be able to find enough information in this Handbook to judge if it is worthwhile to pursue the quest. In addition to the primary information given in the Handbook, there are references to relevant articles, books or lecture notes to help the reader. An excellent index has been included which is extensive and not limited to definitions, theorems etc. The Handbook of Algebra will publish articles as they are received and thus the reader will find in this third volume articles from twelve different sections. The advantages of this scheme are two-fold: accepted articles will be published quickly and the outline of the Handbook can be allowed to evolve as the various volumes are published. A particularly important function of the Handbook is to provide professional mathematicians working in an area other than their own with sufficient information on the topic in question if and when it is needed.- Thorough and practical source of information - Provides in-depth coverage of new topics in algebra - Includes references to relevant articles, books and lecture notes

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