lurie higher algebra

Delving into the Depths: A Comprehensive Guide to Lurie's Higher Algebra

Introduction:

Are you ready to embark on a journey into the fascinating world of higher algebra? This isn't your typical undergraduate algebra course. We're diving headfirst into the groundbreaking work of Jacob Lurie, specifically his monumental tome, "Higher Algebra." This comprehensive guide will demystify the complexities of Lurie's approach, offering a roadmap for navigating this challenging but ultimately rewarding subject. We'll explore the key concepts, provide context for understanding its significance, and offer resources to help you on your path. Prepare to unravel the intricacies of ∞-categories, higher categories, and their profound applications in diverse areas of mathematics. This post serves as your essential starting point, breaking down the intimidating reputation of Lurie's work and highlighting its elegance and power.

I. What is "Higher Algebra" and Why Should You Care?

Traditional algebra deals primarily with sets and functions between them. Lurie's "Higher Algebra," however, elevates this perspective, introducing the concept of ∞-categories. Instead of sets, we now work with objects that possess an intricate web of higher-dimensional morphisms – morphisms between morphisms, morphisms between those morphisms, and so on, ad infinitum. This seemingly abstract shift has profound consequences.

Why is this important? Higher algebra provides a powerful framework for unifying various seemingly disparate areas of mathematics. It offers a more flexible and nuanced language for describing intricate structures, making it an invaluable tool in fields like:

Homological Algebra: Higher algebra provides a streamlined approach to understanding derived categories and their generalizations.
Algebraic Topology: The language of ∞-categories is naturally suited to describe the intricate relationships between topological spaces and their algebraic invariants.
Representation Theory: Higher algebraic methods offer new perspectives on representation theory, particularly in the context of infinite-dimensional representations.
Mathematical Physics: ∞-categories provide a powerful language for describing quantum field theories and other structures relevant to theoretical physics.

The shift to higher categories allows for a more refined understanding of concepts like localization and limits, leading to cleaner proofs and a deeper understanding of underlying structures.

II. Key Concepts in Lurie's Higher Algebra

Understanding Lurie's work requires grappling with several fundamental concepts:

∞-categories: These are categories where morphisms can compose in higher dimensions. This richer structure allows for capturing subtle relationships that are lost in traditional category theory. Think of them as categories "with extra structure" allowing for a more sophisticated understanding of composition.

(∞,1)-categories: A special type of ∞-category where higher morphisms are invertible. This simplification makes many calculations significantly easier while retaining the essential features of higher categorical structures.

Simplicial Sets: Lurie uses simplicial sets as a model for ∞-categories, providing a concrete way to work with these abstract structures. Understanding the basics of simplicial set theory is crucial for navigating the technical aspects of the book.

Derived Algebraic Geometry: Lurie's work has significantly impacted derived algebraic geometry, providing a robust framework for studying algebraic varieties in a more sophisticated context.

III. Navigating the Labyrinth: Structure and Approach

Lurie's "Higher Algebra" is not a casual read. Its rigorous treatment and advanced mathematical language demand a significant level of preparation. However, a strategic approach can make the journey more manageable.

A. Prerequisites:

A solid foundation in category theory, homological algebra, and algebraic topology is essential. Familiarity with basic concepts like limits, colimits, and derived functors is crucial.

B. Structure of the Book:

The book is organized into several chapters, each delving deeper into specific aspects of higher algebra. While the exact structure may evolve with revisions, a general outline typically includes:

Introduction: Establishes the foundational concepts and sets the stage for the subsequent chapters.
∞-categories: A detailed exploration of the theory of ∞-categories, their properties, and various models.
Higher Topoi: Development of the theory of higher topoi, which serve as the appropriate setting for much of higher algebraic geometry.
Sheaves of ∞-categories: Discusses how ∞-categories can be considered as "spaces" and how sheaves of ∞-categories capture important information about these spaces.
Spectral Algebraic Geometry: The culmination of the theoretical framework applied to the study of algebraic varieties.

IV. Resources and Further Exploration

Overcoming the challenges of Lurie's "Higher Algebra" requires a multi-faceted approach. Supplementing the book with other resources is highly recommended:

Lectures and Courses: Seek out online lectures or university courses that cover similar topics. Many universities offer advanced courses utilizing Lurie's framework.
Study Groups: Collaborating with other mathematicians can significantly enhance your understanding. Forming or joining a study group provides a supportive environment for tackling difficult concepts.
Online Forums and Communities: Engage with online communities dedicated to higher algebra. These platforms provide opportunities to ask questions and engage in discussions with other learners.

V. A Hypothetical Textbook Outline: "A Gentle Introduction to Lurie's Higher Algebra"

This hypothetical textbook aims to provide a more accessible approach to Lurie's ideas:

Introduction: Motivation, overview of traditional algebra, and the need for a higher-categorical approach.
Chapter 1: Category Theory Refresher: A review of essential categorical concepts, focusing on those crucial for higher algebra.
Chapter 2: Introduction to ∞-categories: A less technically demanding introduction to ∞-categories using intuitive examples and visualizations.
Chapter 3: Simplicial Sets and ∞-categories: Building a bridge between the abstract notion of ∞-categories and the concrete model of simplicial sets.
Chapter 4: Key Constructions in ∞-Categories: Exploring fundamental concepts like limits, colimits, and adjunctions in the context of ∞-categories.
Chapter 5: Applications to Homological Algebra: Demonstrating the power of ∞-categories in simplifying and unifying aspects of homological algebra.
Chapter 6: A Glimpse into Derived Algebraic Geometry: Introducing the key ideas of derived algebraic geometry without delving into excessive technical details.
Conclusion: Summarizing key takeaways and highlighting open problems and directions for future research.

VI. Expanding on the Hypothetical Textbook Outline Points:

(Introduction): This section would set the stage by revisiting familiar algebraic concepts and motivating the necessity for a higher categorical perspective. Simple examples would illustrate the limitations of traditional algebra when dealing with complex structures.

(Chapter 1): A thorough review of category theory, emphasizing functors, natural transformations, limits, colimits, and adjunctions, all explained with clear examples and visualizations.

(Chapter 2): This chapter would introduce the concept of ∞-categories intuitively, avoiding overly formal definitions initially. Emphasis would be on building an understanding of the core idea rather than getting bogged down in technicalities.

(Chapter 3): A gentler approach to simplicial sets, focusing on their intuitive aspects before progressing to more formal definitions and constructions. The chapter would emphasize the connection between simplicial sets and ∞-categories.

(Chapter 4): A careful and well-illustrated explanation of fundamental constructions in ∞-categories, emphasizing the similarities and differences compared to traditional category theory. Examples and intuitive explanations would be used to clarify abstract concepts.

(Chapter 5): This chapter would showcase the power of ∞-categories by demonstrating how they simplify and unify various aspects of homological algebra. Specific examples would be drawn from derived categories and related concepts.

(Chapter 6): A taste of derived algebraic geometry, focusing on the intuitive aspects and avoiding highly technical details. The goal would be to convey the essential ideas and demonstrate the importance of ∞-categories in this field.

(Conclusion): A concise summary of the key concepts and a broader overview of the significance of Lurie's work within the broader landscape of mathematics.

VII. FAQs:

1. Is Lurie's "Higher Algebra" self-taught? Highly challenging to self-teach due to the complexity and advanced prerequisites. A structured course or mentorship is strongly advised.

1. What is the best way to approach learning this material? Start with solid grounding in category theory and homological algebra. Work through simpler introductions to ∞-categories before tackling Lurie's book.

1. Are there alternative resources besides Lurie's book? Yes, numerous research papers, lecture notes, and online courses explore related concepts.

1. How long does it take to learn Lurie's higher algebra? It’s a long-term commitment, requiring years of dedicated study, especially for complete mastery.

1. What programming languages are used in this field? While not directly programming-focused, familiarity with tools like Python for symbolic computation can be helpful.

1. What are the career prospects for someone proficient in higher algebra? Excellent prospects in academia and research roles, and growing applications in theoretical computer science and physics.

1. Is there an easy-to-understand summary of Lurie's work? No single summary can capture the full depth, but introductory texts and online resources aim to simplify key concepts.

1. What are some common misconceptions about higher algebra? That it's purely abstract with no applications; in reality, it has profound implications in various fields.

1. Are there any open problems in higher algebra? Yes, many open questions remain, driving ongoing research and development in this active area of mathematics.

VIII. Related Articles:

1. An Introduction to Category Theory: A foundational overview of the essential concepts in category theory.
2. Homological Algebra: A Gentle Introduction: A simpler explanation of the core concepts of homological algebra.
3. Simplicial Sets: A Primer: An accessible introduction to the basics of simplicial set theory.
4. Derived Categories Explained: A clear explanation of derived categories and their applications.
5. ∞-Categories: A Visual Approach: A more intuitive introduction to ∞-categories using diagrams and visualizations.
6. The Role of ∞-Categories in Algebraic Topology: Exploring the applications of ∞-categories in algebraic topology.
7. A Beginner's Guide to Derived Algebraic Geometry: A simplified overview of the key ideas in derived algebraic geometry.
8. Applications of Higher Algebra in Mathematical Physics: Exploring the uses of higher

algebra in various physics theories.

9. The Future of Higher Algebra: Discussing potential future research directions and applications within the field.

lurie higher algebra: *Higher Topos Theory* Jacob Lurie, 2009-07-26 In 'Higher Topos Theory', Jacob Lurie presents the foundations of this theory using the language of weak Kan complexes introduced by Boardman and Vogt, and shows how existing theorems in algebraic topology can be reformulated and generalized in the theory's new language.

lurie higher algebra: Higher Categories and Homotopical Algebra Denis-Charles Cisinski, 2019-05-02 At last, a friendly introduction to modern homotopy theory after Joyal and Lurie, reaching advanced tools and starting from scratch.

lurie higher algebra: Weil's Conjecture for Function Fields Dennis Gaitsgory, Jacob Lurie, 2019-02-19 A central concern of number theory is the study of local-to-global principles, which describe the behavior of a global field K in terms of the behavior of various completions of K. This book looks at a specific example of a local-to-global principle: Weil’s conjecture on the Tamagawa number of a semisimple algebraic group G over K. In the case where K is the function field of an algebraic curve X, this conjecture counts the number of G-bundles on X (global information) in terms of the reduction of G at the points of X (local information). The goal of this book is to give a conceptual proof of Weil’s conjecture, based on the geometry of the moduli stack of G-bundles. Inspired by ideas from algebraic topology, it introduces a theory of factorization homology in the setting l-adic sheaves. Using this theory, Dennis Gaitsgory and Jacob Lurie articulate a different local-to-global principle: a product formula that expresses the cohomology of the moduli stack of G-bundles (a global object) as a tensor product of local factors. Using a version of the Grothendieck-Lefschetz trace formula, Gaitsgory and Lurie show that this product formula implies Weil’s conjecture. The proof of the product formula will appear in a sequel volume.

lurie higher algebra: Elements of ∞-Category Theory Emily Riehl, Dominic Verity, 2022-02-10 The language of ∞-categories provides an insightful new way of expressing many results in higher-dimensional mathematics but can be challenging for the uninitiated. To explain what exactly an ∞-category is requires various technical models, raising the question of how they might be compared. To overcome this, a model-independent approach is desired, so that theorems proven with any model would apply to them all. This text develops the theory of ∞-categories from first principles in a model-independent fashion using the axiomatic framework of an ∞-cosmos, the universe in which ∞-categories live as objects. An ∞-cosmos is a fertile setting for the formal category theory of ∞-categories, and in this way the foundational proofs in ∞-category theory closely resemble the classical foundations of ordinary category theory. Equipped with exercises and appendices with background material, this first introduction is meant for students and researchers who have a strong foundation in classical 1-category theory.

lurie higher algebra: Modern Classical Homotopy Theory Jeffrey Strom, 2011-10-19 The core of classical homotopy theory is a body of ideas and theorems that emerged in the 1950s and was later largely codified in the notion of a model category. This core includes the notions of fibration and cofibration; CW complexes; long fiber and cofiber sequences; loop spaces and suspensions; and so on. Brown's representability theorems show that homology and cohomology are also contained in classical homotopy theory. This text develops classical homotopy theory from a modern point of view, meaning that the exposition is informed by the theory of model categories and that homotopy limits and colimits play central roles. The exposition is guided by the principle that it is generally preferable to prove topological results using topology (rather than algebra). The language and basic theory of homotopy limits and colimits make it possible to penetrate deep into the subject with just the rudiments of algebra. The text does reach advanced territory, including the Steenrod algebra, Bott periodicity, localization, the Exponent Theorem of Cohen, Moore, and Neisendorfer, and Miller's Theorem on the Sullivan Conjecture. Thus the reader is given the tools needed to understand and participate in research at (part of) the current frontier of homotopy theory. Proofs are not provided outright. Rather, they are presented in the form of directed problem sets. To the expert, these read as terse proofs; to novices they are challenges that draw them in and help them to thoroughly understand the arguments.

lurie higher algebra: Structured Ring Spectra Andrew Baker, Birgit Richter, 2004-11-18 This book contains some important new contributions to the theory of structured ring spectra.

lurie higher algebra: Categories for the Working Mathematician Saunders Mac Lane, 2013-04-17 An array of general ideas useful in a wide variety of fields. Starting from the foundations, this book illuminates the concepts of category, functor, natural transformation, and duality. It then turns to adjoint functors, which provide a description of universal constructions, an analysis of the representations of functors by sets of morphisms, and a means of manipulating direct and inverse limits. These categorical concepts are extensively illustrated in the remaining chapters, which include many applications of the basic existence theorem for adjoint functors. The categories of algebraic systems are constructed from certain adjoint-like data and characterised by Beck's theorem. After considering a variety of applications, the book continues with the construction and exploitation of Kan extensions. This second edition includes a number of revisions and additions, including new chapters on topics of active interest: symmetric monoidal categories and braided monoidal categories, and the coherence theorems for them, as well as 2-categories and the higher dimensional categories which have recently come into prominence.

lurie higher algebra: *Towards Higher Categories* John C. Baez, J. Peter May, 2009-09-24 The purpose of this book is to give background for those who would like to delve into some higher category theory. It is not a primer on higher category theory itself. It begins with a paper by John Baez and Michael Shulman which explores informally, by analogy and direct connection, how cohomology and other tools of algebraic topology are seen through the eyes of n-category theory. The idea is to give some of the motivations behind this subject. There are then two survey articles, by Julie Bergner and Simona Paoli, about (infinity,1) categories and about the algebraic modelling of homotopy n-types. These are areas that are particularly well understood, and where a fully integrated theory exists. The main focus of the book is on the richness to be found in the theory of bicategories, which gives the essential starting point towards the understanding of higher categorical structures. An article by Stephen Lack gives a thorough, but informal, guide to this theory. A paper by Larry Breen on the theory of gerbes shows how such categorical structures appear in differential geometry. This book is dedicated to Max Kelly, the founder of the Australian school of category theory, and an historical paper by Ross Street describes its development.

lurie higher algebra: Categorical Homotopy Theory Emily Riehl, 2014-05-26 This book develops abstract homotopy theory from the categorical perspective with a particular focus on examples. Part I discusses two competing perspectives by which one typically first encounters homotopy (co)limits: either as derived functors definable when the appropriate diagram categories admit a compatible model structure, or through particular formulae that give the right notion in certain examples. Emily Riehl unifies these seemingly rival perspectives and demonstrates that model structures on diagram categories are irrelevant. Homotopy (co)limits are explained to be a special case of weighted (co)limits, a foundational topic in enriched category theory. In Part II, Riehl further examines this topic, separating categorical arguments from homotopical ones. Part III treats the most ubiquitous axiomatic framework for homotopy theory - Quillen's model categories. Here, Riehl simplifies familiar model categorical lemmas and definitions by focusing on weak factorization systems. Part IV introduces quasi-categories and homotopy coherence.

lurie higher algebra: *Mathematics without Apologies* Michael Harris, 2017-05-30 An insightful reflection on the mathematical soul What do pure mathematicians do, and why do they do it?

Looking beyond the conventional answers—for the sake of truth, beauty, and practical applications—this book offers an eclectic panorama of the lives and values and hopes and fears of mathematicians in the twenty-first century, assembling material from a startlingly diverse assortment of scholarly, journalistic, and pop culture sources. Drawing on his personal experiences and obsessions as well as the thoughts and opinions of mathematicians from Archimedes and Omar Khayyám to such contemporary giants as Alexander Grothendieck and Robert Langlands, Michael Harris reveals the charisma and romance of mathematics as well as its darker side. In this portrait of mathematics as a community united around a set of common intellectual, ethical, and existential challenges, he touches on a wide variety of questions, such as: Are mathematicians to blame for the 2008 financial crisis? How can we talk about the ideas we were born too soon to understand? And how should you react if you are asked to explain number theory at a dinner party? Disarmingly candid, relentlessly intelligent, and richly entertaining, Mathematics without Apologies takes readers on an unapologetic guided tour of the mathematical life, from the philosophy and sociology of mathematics to its reflections in film and popular music, with detours through the mathematical and mystical traditions of Russia, India, medieval Islam, the Bronx, and beyond.

lurie higher algebra: Homotopical Algebraic Geometry II: Geometric Stacks and Applications Bertrand Toën, Gabriele Vezzosi, 2008 This is the second part of a series of papers called HAG, devoted to developing the foundations of homotopical algebraic geometry. The authors start by defining and studying generalizations of standard notions of linear algebra in an abstract monoidal model category, such as derivations, étale and smooth morphisms, flat and projective modules, etc. They then use their theory of stacks over model categories to define a general notion of geometric stack over a base symmetric monoidal model category C, and prove that this notion satisfies the expected properties.

lurie higher algebra: Motivic Homotopy Theory Bjorn Ian Dundas, Marc Levine, P.A. Østvær, Oliver Röndigs, Vladimir Voevodsky, 2007-07-11 This book is based on lectures given at a summer school on motivic homotopy theory at the Sophus Lie Centre in Nordfjordeid, Norway, in August 2002. Aimed at graduate students in algebraic topology and algebraic geometry, it contains background material from both of these fields, as well as the foundations of motivic homotopy theory. It will serve as a good introduction as well as a convenient reference for a broad group of mathematicians to this important and fascinating new subject. Vladimir Voevodsky is one of the founders of the theory and received the Fields medal for his work, and the other authors have all done important work in the subject.

lurie higher algebra: Simplicial Homotopy Theory Paul G. Goerss, John F. Jardine, 2012-12-06 Since the beginning of the modern era of algebraic topology, simplicial methods have been used systematically and effectively for both computation and basic theory. With the development of Quillen's concept of a closed model category and, in particular, a simplicial model category, this collection of methods has become the primary way to describe non-abelian homological algebra and to address homotopy-theoretical issues in a variety of fields, including algebraic K-theory. This book supplies a modern exposition of these ideas, emphasizing model category theoretical techniques. Discussed here are the homotopy theory of simplicial sets, and other basic topics such as simplicial groups, Postnikov towers, and bisimplicial sets. The more advanced material includes homotopy limits and colimits, localization with respect to a map and with respect to a homology theory, cosimplicial spaces, and homotopy coherence. Interspersed throughout are many results and ideas well-known to experts, but uncollected in the literature. Intended for second-year graduate students and beyond, this book introduces many of the basic tools of modern homotopy theory. An extensive background in topology is not assumed.

lurie higher algebra: Homotopy Theory of Higher Categories Carlos Simpson, 2011-10-20 The study of higher categories is attracting growing interest for its many applications in topology, algebraic geometry, mathematical physics and category theory. In this highly readable book, Carlos Simpson develops a full set of homotopical algebra techniques and proposes a working theory of higher categories. Starting with a cohesive overview of the many different approaches currently used by researchers, the author proceeds with a detailed exposition of one of the most widely used techniques: the construction of a Cartesian Quillen model structure for higher categories. The fully iterative construction applies to enrichment over any Cartesian model category, and yields model categories for weakly associative n-categories and Segal n-categories. A corollary is the construction of higher functor categories which fit together to form the (n+1)-category of n-categories. The approach uses Tamsamani's definition based on Segal's ideas, iterated as in Pelissier's thesis using modern techniques due to Barwick, Bergner, Lurie and others.

lurie higher algebra: Category Theory in Context Emily Riehl, 2017-03-09 Introduction to concepts of category theory — categories, functors, natural transformations, the Yoneda lemma, limits and colimits, adjunctions, monads — revisits a broad range of mathematical examples from the categorical perspective. 2016 edition.

lurie higher algebra: Colored Operads Donald Yau, 2016-02-29 The subject of this book is the theory of operads and colored operads, sometimes called symmetric multicategories. A (colored) operad is an abstract object which encodes operations with multiple inputs and one output and relations between such operations. The theory originated in the early 1970s in homotopy theory and quickly became very important in algebraic topology, algebra, algebraic geometry, and even theoretical physics (string theory). Topics covered include basic graph theory, basic category theory, colored operads, and algebras over colored operads. Free colored operads are discussed in complete detail and in full generality. The intended audience of this book includes students and researchers in mathematics and other sciences where operads and colored operads are used. The prerequisite for this book is minimal. Every major concept is thoroughly motivated. There are many graphical illustrations and about 150 exercises. This book can be used in a graduate course and for independent study.

lurie higher algebra: Rings and Their Modules Paul E. Bland, 2011 This book is an introduction to the theory of rings and modules that goes beyond what one normally obtains in a graduate course in abstract algebra. In addition to the presentation of standard topics in ring and module theory, it also covers category theory, homological algebra and even more specialized topics like injective envelopes and proj

lurie higher algebra: Higher Operads, Higher Categories Tom Leinster, 2004-07-22 Foundations of higher dimensional category theory for graduate students and researchers in mathematics and mathematical physics.

lurie higher algebra: Revisiting the de Rham-Witt Complex Bhargav Bhatt, Jacob Lurie, Akhil Mathew, 2021

lurie higher algebra: Lectures on Field Theory and Topology Daniel S. Freed, 2019-08-23 These lectures recount an application of stable homotopy theory to a concrete problem in low energy physics: the classification of special phases of matter. While the joint work of the author and Michael Hopkins is a focal point, a general geometric frame of reference on quantum field theory is emphasized. Early lectures describe the geometric axiom systems introduced by Graeme Segal and Michael Atiyah in the late 1980s, as well as subsequent extensions. This material provides an entry point for mathematicians to delve into quantum field theory. Classification theorems in low dimensions are proved to illustrate the framework. The later lectures turn to more specialized topics in field theory, including the relationship between invertible field theories and stable homotopy theory, extended unitarity, anomalies, and relativistic free fermion systems. The accompanying mathematical explanations touch upon (higher) category theory, duals to the sphere spectrum, equivariant spectra, differential cohomology, and Dirac operators. The outcome of computations made using the Adams spectral sequence is presented and compared to results in the condensed matter literature obtained by very different means. The general perspectives and specific applications fuse into a compelling story at the interface of contemporary mathematics and theoretical physics.

lurie higher algebra: Analysis I Terence Tao, 2016-08-29 This is part one of a two-volume book on real analysis and is intended for senior undergraduate students of mathematics who have already been exposed to calculus. The emphasis is on rigour and foundations of analysis. Beginning with the construction of the number systems and set theory, the book discusses the basics of analysis (limits, series, continuity, differentiation, Riemann integration), through to power series, several variable calculus and Fourier analysis, and then finally the Lebesgue integral. These are almost entirely set in the concrete setting of the real line and Euclidean spaces, although there is some material on abstract metric and topological spaces. The book also has appendices on mathematical logic and the decimal system. The entire text (omitting some less central topics) can be taught in two quarters of 25–30 lectures each. The course material is deeply intertwined with the exercises, as it is intended that the student actively learn the material (and practice thinking and writing rigorously) by proving several of the key results in the theory.

lurie higher algebra: *Factorization Algebras in Quantum Field Theory* Kevin Costello, Owen Gwilliam, 2017 This first volume develops factorization algebras with a focus upon examples exhibiting their use in field theory, which will be useful for researchers and graduates.

lurie higher algebra: Alexandre Grothendieck Leila Schneps, 2014 Provides an explanation of what made Alexandre Grothendieck the mathematician that he was. Thirteen articles written by people who knew him personally - some who even studied or collaborated with him over a period of many years - portray Grothendieck at work, explaining the nature of his thought through descriptions of his discoveries and contributions to various subjects, and with impressions, memories, anecdotes, and some biographical elements.

lurie higher algebra: *Higher Segal Spaces* Tobias Dyckerhoff, Mikhail Kapranov, 2019-10-17 This monograph initiates a theory of new categorical structures that generalize the simplicial Segal property to higher dimensions. The authors introduce the notion of a d-Segal space, which is a simplicial space satisfying locality conditions related to triangulations of d-dimensional cyclic polytopes. Focus here is on the 2-dimensional case. Many important constructions are shown to exhibit the 2-Segal property, including Waldhausen’s S-construction, Hecke-Waldhausen constructions, and configuration spaces of flags. The relevance of 2-Segal spaces in the study of Hall and Hecke algebras is discussed. Higher Segal Spaces marks the beginning of a program to systematically study d-Segal spaces in all dimensions d. The elementary formulation of 2-Segal spaces in the opening chapters is accessible to readers with a basic background in homotopy theory. A chapter on Bousfield localizations provides a transition to the general theory, formulated in terms of combinatorial model categories, that features in the main part of the book. Numerous examples throughout assist readers entering this exciting field to move toward active research; established researchers in the area will appreciate this work as a reference.

lurie higher algebra: A Study in Derived Algebraic Geometry Dennis Gaitsgory, Nick Rozenblyum, 2019-12-31 Derived algebraic geometry is a far-reaching generalization of algebraic geometry. It has found numerous applications in various parts of mathematics, most prominently in representation theory. This volume develops the theory of ind-coherent sheaves in the context of derived algebraic geometry. Ind-coherent sheaves are a “renormalization” of quasi-coherent sheaves and provide a natural setting for Grothendieck-Serre duality as well as geometric incarnations of numerous categories of interest in representation theory. This volume consists of three parts and an appendix. The first part is a survey of homotopical algebra in the setting of ∞-categories and the basics of derived algebraic geometry. The second part builds the theory of ind-coherent sheaves as a functor out of the category of correspondences and studies the relationship between ind-coherent and quasi-coherent sheaves. The third part sets up the general machinery of the $\mathrm{(}\infty, 2\mathrm{)}$-category of correspondences needed for the second part. The category of correspondences, via the theory developed in the third part, provides a general framework for Grothendieck's six-functor formalism. The appendix provides the necessary background on $\mathrm{(}\infty, 2\mathrm{)}$-categories needed for the third part.

lurie higher algebra: *Handbook of Homotopy Theory* Haynes Miller, 2020-01-23 The Handbook of Homotopy Theory provides a panoramic view of an active area in mathematics that is currently seeing dramatic solutions to long-standing open problems, and is proving itself of increasing importance across many other mathematical disciplines. The origins of the subject date back to work of Henri Poincaré and Heinz Hopf in the early 20th century, but it has seen enormous progress in the 21st century. A highlight of this volume is an introduction to and diverse applications of the newly established foundational theory of ¥ -categories. The coverage is vast, ranging from axiomatic to applied, from foundational to computational, and includes surveys of applications both geometric and algebraic. The contributors are among the most active and creative researchers in the field. The 22 chapters by 31 contributors are designed to address novices, as well as established mathematicians, interested in learning the state of the art in this field, whose methods are of increasing importance in many other areas.

lurie higher algebra: Homotopy Type Theory: Univalent Foundations of Mathematics ,

lurie higher algebra: *Higher Algebra* Henry Sinclair Hall, Samuel Ratcliffe Knight, 1894

lurie higher algebra: Algebraic Operads Jean-Louis Loday, Bruno Vallette, 2012-08-08 In many areas of mathematics some “higher operations” are arising. These havebecome so important that several research projects refer to such expressions. Higher operationsform new types of algebras. The key to understanding and comparing them, to creating invariants of their action is operad theory. This is a point of view that is 40 years old in algebraic topology, but the new trend is its appearance in several other areas, such as algebraic geometry, mathematical physics, differential geometry, and combinatorics. The present volume is the first comprehensive and systematic approach to algebraic operads. An operad is an algebraic device that serves to study all kinds of algebras (associative, commutative, Lie, Poisson, A-infinity, etc.) from a conceptual point of view. The book presents this topic with an emphasis on Koszul duality theory. After a modern treatment of Koszul duality for associative algebras, the theory is extended to operads. Applications to homotopy algebra are given, for instance the Homotopy Transfer Theorem. Although the necessary notions of algebra are recalled, readers are expected to be familiar with elementary homological algebra. Each chapter ends with a helpful summary and exercises. A full chapter is devoted to examples, and numerous figures are included. After a low-level chapter on Algebra, accessible to (advanced) undergraduate students, the level increases gradually through the book. However, the authors have done their best to make it suitable for graduate students: three appendices review the basic results needed in order to understand the various chapters. Since higher algebra is becoming essential in several research areas like deformation theory, algebraic geometry, representation theory, differential geometry, algebraic combinatorics, and mathematical physics, the book can also be used as a reference work by researchers.

lurie higher algebra: The Homotopy Theory of (∞,1)-Categories Julia E. Bergner, 2018-03-15 The notion of an (∞,1)-category has become widely used in homotopy theory, category theory, and in a number of applications. There are many different approaches to this structure, all of them equivalent, and each with its corresponding homotopy theory. This book provides a relatively self-contained source of the definitions of the different models, the model structure (homotopy theory) of each, and the equivalences between the models. While most of the current literature focusses on how to extend category theory in this context, and centers in particular on the quasi-category model, this book offers a balanced treatment of the appropriate model structures for simplicial categories, Segal categories, complete Segal spaces, quasi-categories, and relative categories, all from a homotopy-theoretic perspective. Introductory chapters provide background in both homotopy and category theory and contain many references to the literature, thus making the book accessible to graduates and to researchers in related areas.

lurie higher algebra: Topology and Groupoids Ronald Brown, 2006 Annotation. The book is intended as a text for a two-semester course in topology and algebraic topology at the advanced undergraduate orbeginning graduate level. There are over 500 exercises, 114 figures, numerous diagrams. The general direction of the book is towardhomotopy theory with a geometric point of view. This book would providea more than adequate background for a standard algebraic topology coursethat begins with homology theory. For more information seewww.bangor.ac.uk/r.brown/topgpds.htmlThis version dated April 19, 2006, has a number of corrections made.

lurie higher algebra: Fundamental Algebraic Geometry Barbara Fantechi, 2005 Presents an outline of Alexander Grothendieck's theories. This book discusses four main themes - descent theory, Hilbert and Quot schemes, the formal existence theorem, and the Picard scheme. It is suitable for those working in algebraic geometry.

lurie higher algebra: Algebraic Geometry Robin Hartshorne, 2013-06-29 An introduction to abstract algebraic geometry, with the only prerequisites being results from commutative algebra, which are stated as needed, and some elementary topology. More than 400 exercises distributed throughout the book offer specific examples as well as more specialised topics not treated in the main text, while three appendices present brief accounts of some areas of current research. This book can thus be used as textbook for an introductory course in algebraic geometry following a basic graduate course in algebra. Robin Hartshorne studied algebraic geometry with Oscar Zariski and David Mumford at Harvard, and with J.-P. Serre and A. Grothendieck in Paris. He is the author of Residues and Duality, Foundations of Projective Geometry, Ample Subvarieties of Algebraic Varieties, and numerous research titles.

lurie higher algebra: Basic Abstract Algebra Robert B. Ash, 2013-06-17 Relations between groups and sets, results and methods of abstract algebra in terms of number theory and geometry, and noncommutative and homological algebra. Solutions. 2006 edition.

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