

KAC MOODY ALGEBRA

DELVING INTO THE DEPTHS OF KAC-MOODY ALGEBRAS: A COMPREHENSIVE GUIDE

INTRODUCTION:

ARE YOU INTRIGUED BY THE FASCINATING WORLD OF INFINITE-DIMENSIONAL LIE ALGEBRAS? DO TERMS LIKE "CARTAN MATRICES," "AFFINE ALGEBRAS," AND "ROOT SYSTEMS" PIQUE YOUR CURIOSITY? THEN PREPARE TO EMBARK ON A JOURNEY INTO THE CAPTIVATING REALM OF KAC-MOODY ALGEBRAS! THIS COMPREHENSIVE GUIDE WILL UNRAVEL THE MYSTERIES OF THIS POWERFUL MATHEMATICAL STRUCTURE, PROVIDING A DETAILED EXPLORATION SUITABLE FOR BOTH BEGINNERS AND THOSE WITH PRIOR KNOWLEDGE OF LIE ALGEBRAS. WE'LL DELVE INTO THEIR FUNDAMENTAL CONCEPTS, EXPLORE THEIR DIVERSE APPLICATIONS, AND UNCOVER THE ELEGANCE AND POWER BEHIND THEIR INTRICATE STRUCTURE. PREPARE TO UNLOCK THE SECRETS OF KAC-MOODY ALGEBRAS!

1. UNDERSTANDING THE FUNDAMENTALS: WHAT ARE KAC-MOODY ALGEBRAS?

KAC-MOODY ALGEBRAS REPRESENT A SIGNIFICANT GENERALIZATION OF FINITE-DIMENSIONAL SEMISIMPLE LIE ALGEBRAS. WHILE FINITE-DIMENSIONAL LIE ALGEBRAS HAVE A FINITE NUMBER OF GENERATORS AND RELATIONS, KAC-MOODY ALGEBRAS EXTEND THIS FRAMEWORK TO ENCOMPASS INFINITE-DIMENSIONAL STRUCTURES. THIS GENERALIZATION IS ACHIEVED THROUGH THE INTRODUCTION OF A GENERALIZED CARTAN MATRIX, A MATRIX OF INTEGERS THAT ENCODES THE COMMUTATION RELATIONS BETWEEN THE GENERATORS OF THE ALGEBRA. THIS CARTAN MATRIX DICTATES THE ALGEBRA'S STRUCTURE, INFLUENCING ITS ROOT SYSTEM, REPRESENTATIONS, AND OVERALL BEHAVIOR. UNLIKE FINITE-DIMENSIONAL LIE ALGEBRAS WHERE THE CARTAN MATRIX IS POSITIVE DEFINITE, KAC-MOODY ALGEBRAS ALLOW FOR MORE GENERAL, POSSIBLY INDEFINITE, CARTAN MATRICES. THIS SEEMINGLY MINOR CHANGE OPENS UP A VAST LANDSCAPE OF NEW MATHEMATICAL POSSIBILITIES.

2. THE GENERALIZED CARTAN MATRIX: THE HEART OF THE MATTER

THE GENERALIZED CARTAN MATRIX IS THE KEYSTONE OF ANY KAC-MOODY ALGEBRA. IT'S A SQUARE MATRIX, TYPICALLY DENOTED BY A , WHERE THE ENTRIES A_{ij} DICTATE THE COMMUTATION RELATIONS BETWEEN THE GENERATORS OF THE ALGEBRA. THE DIAGONAL ENTRIES ARE ALWAYS 2, WHILE OFF-DIAGONAL ENTRIES ARE NON-POSITIVE INTEGERS. THE PROPERTIES OF THIS MATRIX, PARTICULARLY ITS RANK AND THE NATURE OF ITS EIGENVALUES, DETERMINE THE TYPE OF KAC-MOODY ALGEBRA BEING CONSIDERED. FOR INSTANCE, IF THE MATRIX IS POSITIVE DEFINITE, WE HAVE A FINITE-DIMENSIONAL SEMISIMPLE LIE ALGEBRA. IF IT IS POSITIVE SEMI-DEFINITE WITH A ONE-DIMENSIONAL NULL SPACE, WE HAVE AN AFFINE KAC-MOODY ALGEBRA, A CRUCIAL CLASS WITH SIGNIFICANT APPLICATIONS IN PHYSICS. OTHER TYPES INCLUDE INDEFINITE KAC-MOODY ALGEBRAS, EXHIBITING MUCH RICHER AND MORE COMPLEX STRUCTURES. UNDERSTANDING THE NUANCES OF THE CARTAN MATRIX IS PARAMOUNT TO GRASPING THE INTRICACIES OF THE CORRESPONDING KAC-MOODY ALGEBRA.

3. ROOT SYSTEMS AND REPRESENTATIONS: UNVEILING THE ALGEBRA'S STRUCTURE

THE ROOT SYSTEM OF A KAC-MOODY ALGEBRA IS A CRUCIAL ASPECT OF ITS STRUCTURE. JUST AS IN FINITE-DIMENSIONAL LIE ALGEBRAS, THE ROOTS ARE VECTORS THAT DESCRIBE THE WEIGHTS OF IRREDUCIBLE REPRESENTATIONS. HOWEVER, IN THE KAC-MOODY CONTEXT, THE ROOT SYSTEM CAN BE SIGNIFICANTLY MORE COMPLEX AND INFINITE. THE ROOT SYSTEM'S STRUCTURE, INCLUDING THE PRESENCE OF REAL AND IMAGINARY ROOTS, IS DIRECTLY DICTATED BY THE CARTAN MATRIX. THE REPRESENTATION THEORY OF KAC-MOODY ALGEBRAS IS SIMILARLY RICH AND COMPLEX, PROVIDING A POWERFUL TOOL FOR ANALYZING THEIR STRUCTURE AND APPLICATIONS. UNDERSTANDING THE REPRESENTATIONS ALLOWS FOR THE CLASSIFICATION AND ANALYSIS OF VARIOUS PHYSICAL SYSTEMS AND MATHEMATICAL STRUCTURES.

4. AFFINE KAC-MOODY ALGEBRAS: A SPECIAL CASE WITH PROFOUND SIGNIFICANCE

AFFINE KAC-MOODY ALGEBRAS REPRESENT A PARTICULARLY IMPORTANT CLASS WITHIN THE BROADER FAMILY OF KAC-MOODY ALGEBRAS. THESE ALGEBRAS ARE CHARACTERIZED BY A CARTAN MATRIX THAT IS POSITIVE SEMI-DEFINITE WITH A ONE-DIMENSIONAL NULL SPACE. THIS SEEMINGLY SUBTLE DIFFERENCE HAS PROFOUND IMPLICATIONS. AFFINE KAC-MOODY ALGEBRAS HAVE A RICH AND WELL-UNDERSTOOD STRUCTURE, MAKING THEM AMENABLE TO DETAILED ANALYSIS. FURTHERMORE, THEY HAVE FOUND NUMEROUS APPLICATIONS IN VARIOUS AREAS OF PHYSICS, INCLUDING CONFORMAL FIELD THEORY AND STRING THEORY. THEIR APPEARANCE IN THESE CONTEXTS UNDERSCORES THEIR FUNDAMENTAL ROLE IN MODERN THEORETICAL PHYSICS.

5. APPLICATIONS OF KAC-MOODY ALGEBRAS: BEYOND THE THEORETICAL

THE THEORETICAL ELEGANCE OF KAC-MOODY ALGEBRAS EXTENDS TO REMARKABLE APPLICATIONS ACROSS MULTIPLE DISCIPLINES. THEIR IMPACT IS FELT MOST SIGNIFICANTLY IN:

CONFORMAL FIELD THEORY: AFFINE KAC-MOODY ALGEBRAS PROVIDE A POWERFUL FRAMEWORK FOR UNDERSTANDING CONFORMAL FIELD THEORIES, WHICH PLAY A PIVOTAL ROLE IN STRING THEORY AND STATISTICAL MECHANICS.

STRING THEORY: THESE ALGEBRAS ARE DEEPLY INTERTWINED WITH STRING THEORY, APPEARING IN THE DESCRIPTION OF THE SYMMETRIES AND INTERACTIONS OF STRINGS.

INTEGRABLE SYSTEMS: THEIR STRUCTURE UNDERPINS THE MATHEMATICAL UNDERSTANDING OF MANY INTEGRABLE SYSTEMS, FINDING APPLICATIONS IN VARIOUS AREAS OF PHYSICS AND MATHEMATICS.

REPRESENTATION THEORY: KAC-MOODY ALGEBRAS HAVE GREATLY ENRICHED REPRESENTATION THEORY, PROVIDING NEW AND POWERFUL TECHNIQUES FOR UNDERSTANDING THE REPRESENTATIONS OF INFINITE-DIMENSIONAL LIE ALGEBRAS.

6. EXPLORING ADVANCED TOPICS: BEYOND THE BASICS

FOR THOSE SEEKING A DEEPER DIVE INTO THE INTRICACIES OF KAC-MOODY ALGEBRAS, SEVERAL ADVANCED TOPICS AWAIT EXPLORATION:

WEYL GROUPS AND THEIR PROPERTIES: UNDERSTANDING THE WEYL GROUP, A GROUP OF REFLECTIONS GENERATED BY THE SIMPLE ROOTS, IS ESSENTIAL FOR ANALYZING THE ROOT SYSTEM'S STRUCTURE.

HIGHEST WEIGHT REPRESENTATIONS: A DETAILED STUDY OF HIGHEST WEIGHT REPRESENTATIONS PROVIDES VALUABLE INSIGHT INTO THE REPRESENTATION THEORY OF KAC-MOODY ALGEBRAS.

THE KAC-PETERSON FORMULA: THIS FORMULA PROVIDES A POWERFUL TOOL FOR CALCULATING THE CHARACTERS OF IRREDUCIBLE REPRESENTATIONS.

VERTEX OPERATOR ALGEBRAS: VERTEX OPERATOR ALGEBRAS PROVIDE AN ALTERNATIVE, BUT EQUIVALENT, CONSTRUCTION OF AFFINE KAC-MOODY ALGEBRAS.

BOOK OUTLINE: "A JOURNEY INTO KAC-MOODY ALGEBRAS"

I. INTRODUCTION:

WHAT ARE LIE ALGEBRAS?

MOTIVATION FOR KAC-MOODY ALGEBRAS.

OVERVIEW OF THE BOOK'S STRUCTURE.

II. FOUNDATIONS:

DEFINITION AND EXAMPLES OF KAC-MOODY ALGEBRAS.

THE GENERALIZED CARTAN MATRIX AND ITS PROPERTIES.

ROOT SYSTEMS AND THEIR CLASSIFICATION.

III. REPRESENTATION THEORY:

HIGHEST WEIGHT MODULES AND VERMA MODULES.

CHARACTER FORMULAS AND THEIR APPLICATIONS.

INTEGRABLE REPRESENTATIONS.

IV. AFFINE KAC-MOODY ALGEBRAS:

CLASSIFICATION OF AFFINE KAC-MOODY ALGEBRAS.

APPLICATIONS IN CONFORMAL FIELD THEORY.

APPLICATIONS IN STRING THEORY.

V. ADVANCED TOPICS:

WEYL GROUPS AND THEIR ACTIONS.

KAC-PETERSON FORMULA.

VERTEX OPERATOR ALGEBRAS.

VI. CONCLUSION:

SUMMARY OF KEY CONCEPTS.

OPEN PROBLEMS AND FUTURE RESEARCH DIRECTIONS.

(EACH POINT IN THIS OUTLINE WOULD BE EXPANDED UPON IN A FULL BOOK-LENGTH TREATMENT.)

FREQUENTLY ASKED QUESTIONS (FAQs):

1. WHAT IS THE DIFFERENCE BETWEEN A FINITE-DIMENSIONAL LIE ALGEBRA AND A KAC-MOODY ALGEBRA? FINITE-DIMENSIONAL LIE ALGEBRAS HAVE A FINITE NUMBER OF GENERATORS, WHILE KAC-MOODY ALGEBRAS ARE INFINITE-DIMENSIONAL, DEFINED BY A GENERALIZED CARTAN MATRIX.
1. WHAT IS THE SIGNIFICANCE OF THE CARTAN MATRIX? THE CARTAN MATRIX DEFINES THE COMMUTATION RELATIONS BETWEEN THE GENERATORS OF THE KAC-MOODY ALGEBRA AND DICTATES ITS STRUCTURE.
1. WHAT ARE AFFINE KAC-MOODY ALGEBRAS, AND WHY ARE THEY IMPORTANT? AFFINE KAC-MOODY ALGEBRAS ARE A SPECIFIC TYPE OF KAC-MOODY ALGEBRA WITH SIGNIFICANT APPLICATIONS IN CONFORMAL FIELD THEORY AND STRING THEORY.
1. HOW ARE KAC-MOODY ALGEBRAS APPLIED IN PHYSICS? THEY PLAY CRUCIAL ROLES IN CONFORMAL FIELD THEORY, STRING THEORY, AND INTEGRABLE SYSTEMS.
1. WHAT IS THE WEYL GROUP, AND WHAT IS ITS IMPORTANCE? THE WEYL GROUP IS A GROUP OF REFLECTIONS ACTING ON THE ROOT SYSTEM, CRUCIAL FOR UNDERSTANDING THE ROOT SYSTEM'S STRUCTURE.
1. WHAT ARE HIGHEST WEIGHT REPRESENTATIONS? HIGHEST WEIGHT REPRESENTATIONS ARE IRREDUCIBLE REPRESENTATIONS CHARACTERIZED BY A HIGHEST WEIGHT VECTOR.
1. WHAT IS THE KAC-PETERSON FORMULA? IT'S A FORMULA FOR CALCULATING THE CHARACTER OF IRREDUCIBLE REPRESENTATIONS OF KAC-MOODY ALGEBRAS.
1. WHAT ARE VERTEX OPERATOR ALGEBRAS, AND HOW DO THEY RELATE TO KAC-MOODY ALGEBRAS? VERTEX OPERATOR

ALGEBRAS PROVIDE AN ALTERNATIVE CONSTRUCTION OF AFFINE KAC-MOODY ALGEBRAS.

1. ARE THERE ANY UNSOLVED PROBLEMS IN THE STUDY OF KAC-MOODY ALGEBRAS? YES, MANY OPEN PROBLEMS REMAIN, INCLUDING DEEPER UNDERSTANDING OF REPRESENTATIONS AND APPLICATIONS TO OTHER AREAS OF MATHEMATICS AND PHYSICS.

RELATED ARTICLES:

1. LIE ALGEBRAS: A BEGINNER'S GUIDE: A FOUNDATIONAL INTRODUCTION TO THE BASIC CONCEPTS OF LIE ALGEBRAS.
1. CARTAN MATRICES AND THEIR SIGNIFICANCE: A DETAILED EXPLANATION OF THE PROPERTIES AND IMPORTANCE OF CARTAN MATRICES.
1. ROOT SYSTEMS IN LIE ALGEBRAS: AN IN-DEPTH EXPLORATION OF ROOT SYSTEMS AND THEIR CLASSIFICATION.
1. REPRESENTATION THEORY OF FINITE-DIMENSIONAL LIE ALGEBRAS: A REVIEW OF REPRESENTATION THEORY APPLIED TO FINITE-DIMENSIONAL ALGEBRAS.
1. CONFORMAL FIELD THEORY AND ITS APPLICATIONS: AN OVERVIEW OF CONFORMAL FIELD THEORY AND ITS CONNECTION TO KAC-MOODY ALGEBRAS.
1. STRING THEORY AND ITS MATHEMATICAL FOUNDATIONS: AN EXPLORATION OF THE MATHEMATICAL STRUCTURES UNDERLYING STRING THEORY.
1. INTEGRABLE SYSTEMS: A MATHEMATICAL PERSPECTIVE: A DISCUSSION OF INTEGRABLE SYSTEMS AND THEIR RELATIONSHIP TO KAC-MOODY ALGEBRAS.
1. HIGHEST WEIGHT REPRESENTATIONS: A DETAILED STUDY: A DEEPER DIVE INTO THE PROPERTIES AND CHARACTERISTICS OF HIGHEST WEIGHT REPRESENTATIONS.
1. VERTEX OPERATOR ALGEBRAS: CONSTRUCTION AND PROPERTIES: A COMPREHENSIVE EXPLANATION OF VERTEX

Kac moody algebra: Kac-Moody Lie Algebras and Related Topics Neelacanta Sthanumoorthy, Kailash C. Misra, 2004 This volume is the proceedings of the Ramanujan International Symposium on Kac-Moody Lie algebras and their applications. The symposium provided researchers in mathematics and physics with the opportunity to discuss new developments in this rapidly-growing area of research. The book contains several excellent articles with new and significant results. It is suitable for graduate students and researchers working in Kac-Moody Lie algebras, their applications, and related areas of research.

kac moody algebra: Kac-Moody Groups, their Flag Varieties and Representation Theory Shrawan Kumar, 2012-12-06 Kac-Moody Lie algebras were introduced in the mid-1960s independently by V. Kac and R. Moody, generalizing the finite-dimensional semisimple Lie algebras which we refer to as the finite case. The theory has undergone tremendous developments in various directions and connections with diverse areas abound, including mathematical physics, so much so that this theory has become a standard tool in mathematics. A detailed treatment of the Lie algebra aspect of the theory can be found in V. Kac's book [Kac-90] This self-contained work treats the algebro-geometric and the topological aspects of Kac-Moody theory from scratch. The emphasis is on the study of the Kac-Moody groups and their flag varieties XY , including their detailed construction, and their applications to the representation theory of g . In the finite case, g is nothing but a semisimple Y simply-connected algebraic group and X is the flag variety g/P_Y for a parabolic subgroup $p \in C(g)$.

kac moody algebra: Some Generalized Kac-Moody Algebras with Known Root Multiplicities Peter Niemann, 2002 Starting from Borchers' fake monster Lie algebra, this text constructs a sequence of six generalized Kac-Moody algebras whose denominator formulas, root systems and all root multiplicities can be described explicitly. The root systems decompose space into convex holes, of finite and affine type, similar to the situation in the case of the Leech lattice. As a corollary, we obtain strong upper bounds for the root multiplicities of a number of hyperbolic Lie algebras, including AE_{23} .

kac moody algebra: Infinite-Dimensional Lie Algebras Victor G. Kac, 1990 The third, substantially revised edition of a monograph concerned with Kac-Moody algebras, a particular class of infinite-dimensional Lie algebras, and their representations, based on courses given over a number of years at MIT and in Paris.

kac moody algebra: Kac-moody And Virasoro Algebras: A Reprint Volume For Physicists Peter Goddard, David Olive, 1988-06-01 This volume reviews the subject of Kac-Moody and Virasoro Algebras. It serves as a reference book for physicists with commentary notes and reprints.

kac moody algebra: Infinite Dimensional Lie Algebras Victor G. Kac, 2013-11-09

kac moody algebra: Lie Algebras of Finite and Affine Type Roger William Carter, 2005-10-27 This book provides a thorough but relaxed mathematical treatment of Lie algebras.

kac moody algebra: Affine Lie Algebras, Weight Multiplicities, and Branching Rules Sam Kass, R. V. Moody, J. Patera, R. Slansky, 1990-01-01 This practical treatise is an introduction to the mathematics and physics of affine Kac-Moody algebras. It is the result of an unusual interdisciplinary effort by two physicists and two mathematicians to make this field understandable to a broad readership and to illuminate the connections among seemingly disparate domains of mathematics and physics that are tantalizingly suggested by the ubiquity of Lie theory. The book will be useful to Lie algebraists, high energy physicists, statistical mechanics, and number theorists. Volume One contains a description of Kac-Moody Lie algebras, and especially the affine algebras and their representations; the results of extensive computations follow in Volume Two, which is spiral bound for easy reference. This practical treatise is an introduction to the mathematics and physics of

affine Kac-Moody algebras. It is the result of an unusual interdisciplinary effort by two physicists and two mathematicians to make this field understandable to a broad readership and to illuminate the connections among seemingly disparate domains of mathematics and physics that are tantalizingly suggested by the ubiquity of Lie theory. The book will be useful to Lie algebraists, high energy physicists, statistical mechanics, and number theorists. Volume One contains a description of Kac-Moody Lie algebras, and especially the affine algebras and their representations; the results of extensive computations follow in Volume Two, which is spiral bound for easy reference.

kac moody algebra: Lie Algebras, Part 2 E.A. de Kerf, G.G.A. Bäuerle, A.P.E. ten Kroode, 1997-10-30 This is the long awaited follow-up to Lie Algebras, Part I which covered a major part of the theory of Kac-Moody algebras, stressing primarily their mathematical structure. Part II deals mainly with the representations and applications of Lie Algebras and contains many cross references to Part I. The theoretical part largely deals with the representation theory of Lie algebras with a triangular decomposition, of which Kac-Moody algebras and the Virasoro algebra are prime examples. After setting up the general framework of highest weight representations, the book continues to treat topics as the Casimir operator and the Weyl-Kac character formula, which are specific for Kac-Moody algebras. The applications have a wide range. First, the book contains an exposition on the role of finite-dimensional semisimple Lie algebras and their representations in the standard and grand unified models of elementary particle physics. A second application is in the realm of soliton equations and their infinite-dimensional symmetry groups and algebras. The book concludes with a chapter on conformal field theory and the importance of the Virasoro and Kac-Moody algebras therein.

kac moody algebra: Introduction to Kac-Moody Algebra Zhexian Wan, 1991 This book is an introduction to a rapidly growing subject of modern mathematics, the Kac-Moody algebra, which was introduced by V Kac and R Moody simultaneously and independently in 1968.

kac moody algebra: Some Generalized Kac-Moody Algebras with Known Root Multiplicities Nanhua Xi, Peter Niemann, 2014-09-11 Introduction Generalized Kac-Moody algebras Modular forms Lattices and their Theta-functions The proof of Theorem 1.7 The real simple roots Hyperbolic Lie algebras Appendix A Appendix B Bibliography Notation

kac moody algebra: Introduction to Finite and Infinite Dimensional Lie (Super)algebras Neelacanta Sthanumoorthy, 2016-04-26 Lie superalgebras are a natural generalization of Lie algebras, having applications in geometry, number theory, gauge field theory, and string theory. Introduction to Finite and Infinite Dimensional Lie Algebras and Superalgebras introduces the theory of Lie superalgebras, their algebras, and their representations. The material covered ranges from basic definitions of Lie groups to the classification of finite-dimensional representations of semi-simple Lie algebras. While discussing all classes of finite and infinite dimensional Lie algebras and Lie superalgebras in terms of their different classes of root systems, the book focuses on Kac-Moody algebras. With numerous exercises and worked examples, it is ideal for graduate courses on Lie groups and Lie algebras. - Discusses the fundamental structure and all root relationships of Lie algebras and Lie superalgebras and their finite and infinite dimensional representation theory - Closely describes BKM Lie superalgebras, their different classes of imaginary root systems, their complete classifications, root-supermultiplicities, and related combinatorial identities - Includes numerous tables of the properties of individual Lie algebras and Lie superalgebras - Focuses on Kac-Moody algebras

kac moody algebra: Infinite Dimensional Lie Algebras And Groups Victor G Kac, 1989-07-01 Contents: Integrable Representation of Kac-Moody Algebras: Results and Open Problems (V Chari & A Pressley) Existence of Certain Components in the Tensor Product of Two Integrable Highest Weight Modules for Kac-Moody Algebras (SKumar) Frobenius Action on the B-Cohomology (O Mathieu) Certain Rank Two Subsystems of Kac-Moody Root Systems (J Morita) Lie Groups Associated to Kac-Moody Lie Algebras: An Analytic Approach (E Rodriguez-Carrington) Almost Split-K-Forms of Kac-Moody Algebras (G Rousseau) Global Representations of the Diffeomorphism Groups of the Circle (F Bien) Path Space Realization of the Basic Representation of $An(1)$ (E Date et

al)Boson-Fermion Correspondence Over (C De Concini et al)Classification of Modular Invariant Representations of Affine Algebras (V G Kac & M Wakimoto)Standard Monomial Theory for SL_2 (V Lakshmibai & C S Seshadri)Some Results on Modular Invariant Representations (S Lu)Current Algebras in 3+1 Space-Time Dimensions (J Mickelson)Standard Representations of $An(1)$ (M Primc)Representations of the Algebra $U_q(\mathfrak{sl}(2))$, q -Orthogonal Polynomials and Invariants of Links (A N Kirillov & N Yu Reshetikhin)Infinite Super Grassmannians and Super Plücker Equations (M J Bergvelt)Drinfeld-Sokolov Hierarchies and t -Functions (H J Imbens)Super Boson-Fermion Correspondence of Type B (V G Kac & J W van de Leur)Prym Varieties and Soliton Equations (T Shiota)Polynomial Solutions of the BKP Hierarchy and Projective Representations of Symmetric Groups (Y You)Toward Generalized Macdonald's Identities (D Bernard)Conformal Theories with Non-Linearly Extended Virasoro Symmetries and Lie Algebra Classification (A Bilal & J-L Gervais)Extended Conformal Algebras from Kac-Moody Algebras (P Bouwknegt)Meromorphic Conformal Field Theory (P Goddard)Local Extensions of the $U(1)$ Current Algebra and Their Positive Energy Representations (R R Paunov & I T Todorov)Conformal Field Theory on Moduli Family of Stable Curves with Gauge Symmetries (A Tsuchiya & Y Yamada) Readership: Mathematicians and mathematical physicists

kac moody algebra: Langlands Correspondence for Loop Groups Edward Frenkel, 2007-06-28 The first account of local geometric Langlands Correspondence, a new area of mathematical physics developed by the author.

kac moody algebra: An Analogue of a Reductive Algebraic Monoid Whose Unit Group Is a Kac-Moody Group Claus Mokler, 2005 By an easy generalization of the Tannaka-Krein reconstruction we associate to the category of admissible representations of the category $\mathcal{O}$ of a Kac-Moody algebra, and its category of admissible duals, a monoid with a coordinate ring. The Kac-Moody group is the Zariski open dense unit group of this monoid. The restriction of the coordinate ring to the Kac-Moody group is the algebra of strongly regular functions introduced by V. Kac and D. Peterson. This monoid has similar structural properties as a reductive algebraic monoid. In particular it is unit regular, its idempotents related to the faces of the Tits cone. It has Bruhat and Birkhoff decompositions. The Kac-Moody algebra is isomorphic to the Lie algebra of this monoid.

kac moody algebra: Lie Algebras with Triangular Decompositions Robert V. Moody, Arturo Pianzola, 1995-04-17 Imparts a self-contained development of the algebraic theory of Kac-Moody algebras, their representations and close relatives--the Virasoro and Heisenberg algebras. Focuses on developing the theory of triangular decompositions and part of the Kac-Moody theory not specific to the affine case. Also covers lattices, and finite root systems, infinite-dimensional theory, Weyl groups and conjugacy theorems.

kac moody algebra: Finite and Infinite Dimensional Lie Algebras and Applications in Physics Gerard G. A. Bäuerle, 1990

kac moody algebra: Kac-Moody and Virasoro Algebras, 1988

kac moody algebra: Physics and Mathematics of Strings Lars Brink, Daniel Harry Friedan, A. M. Polyakov, 1990 Vadim Knizhnik was one of the most promising theoretical physicists in the world. Unfortunately, he passed away at the very young age of 25 years. This memorial volume is to honor his contributions in Theoretical Physics. This is perhaps one of the most important collections of articles on the theoretical developments in String Theory, Conformal Field Theory and related topics. It consists of contributions from world-renowned physicists who have met Vadim Knizhnik personally and whom the late Knizhnik really respected. The contributions are systematic and pedagogical in format.

kac moody algebra: Introduction To Kac-moody Algebras Zhe-xian Wan, 1991-03-29 This book is an introduction to a rapidly growing subject of modern mathematics, the Kac-Moody algebra, which was introduced by V Kac and R Moody simultaneously and independently in 1968.

kac moody algebra: Group Theory in Physics John F. Cornwell, 1997-07-11 This book, an abridgment of Volumes I and II of the highly respected Group Theory in Physics, presents a carefully

constructed introduction to group theory and its applications in physics. The book provides an introduction to and description of the most important basic ideas and the role that they play in physical problems. The clearly written text contains many pertinent examples that illustrate the topics, even for those with no background in group theory. This work presents important mathematical developments to theoretical physicists in a form that is easy to comprehend and appreciate. Finite groups, Lie groups, Lie algebras, semi-simple Lie algebras, crystallographic point groups and crystallographic space groups, electronic energy bands in solids, atomic physics, symmetry schemes for fundamental particles, and quantum mechanics are all covered in this compact new edition. - Covers both group theory and the theory of Lie algebras - Includes studies of solid state physics, atomic physics, and fundamental particle physics - Contains a comprehensive index - Provides extensive examples

kac moody algebra: Vertex Algebras and Algebraic Curves Edward Frenkel, David Ben-Zvi, 2004-08-25 Vertex algebras are algebraic objects that encapsulate the concept of operator product expansion from two-dimensional conformal field theory. Vertex algebras are fast becoming ubiquitous in many areas of modern mathematics, with applications to representation theory, algebraic geometry, the theory of finite groups, modular functions, topology, integrable systems, and combinatorics. This book is an introduction to the theory of vertex algebras with a particular emphasis on the relationship with the geometry of algebraic curves. The notion of a vertex algebra is introduced in a coordinate-independent way, so that vertex operators become well defined on arbitrary smooth algebraic curves, possibly equipped with additional data, such as a vector bundle. Vertex algebras then appear as the algebraic objects encoding the geometric structure of various moduli spaces associated with algebraic curves. Therefore they may be used to give a geometric interpretation of various questions of representation theory. The book contains many original results, introduces important new concepts, and brings new insights into the theory of vertex algebras. The authors have made a great effort to make the book self-contained and accessible to readers of all backgrounds. Reviewers of the first edition anticipated that it would have a long-lasting influence on this exciting field of mathematics and would be very useful for graduate students and researchers interested in the subject. This second edition, substantially improved and expanded, includes several new topics, in particular an introduction to the Beilinson-Drinfeld theory of factorization algebras and the geometric Langlands correspondence.

kac moody algebra: Abstract" Homomorphisms of Split Kac-Moody Groups" Pierre-Emmanuel Caprace, 2009-03-06 This work is devoted to the isomorphism problem for split Kac-Moody groups over arbitrary fields. This problem turns out to be a special case of a more general problem, which consists in determining homomorphisms of isotropic semisimple algebraic groups to Kac-Moody groups, whose image is bounded. Since Kac-Moody groups possess natural actions on twin buildings, and since their bounded subgroups can be characterized by fixed point properties for these actions, the latter is actually a rigidity problem for algebraic group actions on twin buildings. The author establishes some partial rigidity results, which we use to prove an isomorphism theorem for Kac-Moody groups over arbitrary fields of cardinality at least 4^4 . In particular, he obtains a detailed description of automorphisms of Kac-Moody groups. This provides a complete understanding of the structure of the automorphism group of Kac-Moody groups over ground fields of characteristic 0 . The same arguments allow to treat unitary forms of complex Kac-Moody groups. In particular, the author shows that the Hausdorff topology that these groups carry is an invariant of the abstract group structure. Finally, the author proves the non-existence of cocentral homomorphisms of Kac-Moody groups of indefinite type over infinite fields with finite-dimensional target. This provides a partial solution to the linearity problem for Kac-Moody groups.

kac moody algebra: Vertex Operators in Mathematics and Physics J. Lepowsky, S. Mandelstam, I.M. Singer, 2013-03-08 James Lepowsky The search for symmetry in nature has for a long time provided representation theory with perhaps its chief motivation. According to the standard approach of Lie theory, one looks for infinitesimal symmetry -- Lie algebras of operators or concrete realizations of abstract Lie algebras. A central theme in this volume is the construction of

affine Lie algebras using formal differential operators called vertex operators, which originally appeared in the dual-string theory. Since the precise description of vertex operators, in both mathematical and physical settings, requires a fair amount of notation, we do not attempt it in this introduction. Instead we refer the reader to the papers of Mandelstam, Goddard-Olive, Lepowsky-Wilson and Frenkel-Lepowsky-Meurman. We have tried to maintain consistency of terminology and to some extent notation in the articles herein. To help the reader we shall review some of the terminology. We also thought it might be useful to supplement an earlier fairly detailed exposition of ours [37] with a brief historical account of vertex operators in mathematics and their connection with affine algebras. Since we were involved in the development of the subject, the reader should be advised that what follows reflects our own understanding. For another view, see [29].¹ Partially supported by the National Science Foundation through the Mathematical Sciences Research Institute and NSF Grant MCS 83-01664. ¹ We would like to thank Igor Frenkel for his valuable comments on the first draft of this introduction.

kac moody algebra: Lie Algebras and Their Representations Seok-Jin Kang, Myung-Hwan Kim, Insok Lee, 1996 Over the past 30 years, exciting developments in diverse areas of the theory of Lie algebras and their representations have been observed. The symposium covered topics such as Lie algebras and combinatorics, crystal bases for quantum groups, quantum groups and solvable lattice models, and modular and infinite-dimensional Lie algebras. In this volume, readers will find several excellent expository articles and research papers containing many significant new results in this area.

kac moody algebra: Introduction to Lie Algebras and Representation Theory J.E. Humphreys, 2012-12-06 This book is designed to introduce the reader to the theory of semisimple Lie algebras over an algebraically closed field of characteristic 0, with emphasis on representations. A good knowledge of linear algebra (including eigenvalues, bilinear forms, euclidean spaces, and tensor products of vector spaces) is presupposed, as well as some acquaintance with the methods of abstract algebra. The first four chapters might well be read by a bright undergraduate; however, the remaining three chapters are admittedly a little more demanding. Besides being useful in many parts of mathematics and physics, the theory of semisimple Lie algebras is inherently attractive, combining as it does a certain amount of depth and a satisfying degree of completeness in its basic results. Since Jacobson's book appeared a decade ago, improvements have been made even in the classical parts of the theory. I have tried to incorporate some of them here and to provide easier access to the subject for non-specialists. For the specialist, the following features should be noted: (1) The Jordan-Chevalley decomposition of linear transformations is emphasized, with toral subalgebras replacing the more traditional Cartan subalgebras in the semisimple case. (2) The conjugacy theorem for Cartan subalgebras is proved (following D. J. Winter and G. D. Mostow) by elementary Lie algebra methods, avoiding the use of algebraic geometry.

kac moody algebra: Invariant Measures for Unitary Groups Associated to Kac-Moody Lie Algebras Doug Pickrell, 2000 The main purpose of this paper is to prove the existence, and in some cases the uniqueness, of unitarily invariant measures on formal completions of groups associated to affine Kac-Moody algebras, and associated homogeneous spaces. The basic invariant measure is a natural generalization of Haar measure for a simply connected compact Lie group, and its projection to flag spaces is a generalization of the normalized invariant volume element. The other invariant measures are actually measures having values in line bundles over these spaces; these bundle-valued measures heuristically arise from coupling the basic invariant measure to Hermitian structures on associated line bundles, but in this infinite dimensional setting they are generally singular with respect to the basic invariant measure.

kac moody algebra: Lectures On Infinite-dimensional Lie Algebra Minoru Wakimoto, 2001-10-26 The representation theory of affine Lie algebras has been developed in close connection with various areas of mathematics and mathematical physics in the last two decades. There are three excellent books on it, written by Victor G Kac. This book begins with a survey and review of the material treated in Kac's books. In particular, modular invariance and conformal invariance are

explained in more detail. The book then goes further, dealing with some of the recent topics involving the representation theory of affine Lie algebras. Since these topics are important not only in themselves but also in their application to some areas of mathematics and mathematical physics, the book expounds them with examples and detailed calculations.

kac moody algebra: Infinite Dimensional Groups with Applications Victor Kac, 1985-10-14 This volume records most of the talks given at the Conference on Infinite-dimensional Groups held at the Mathematical Sciences Research Institute at Berkeley, California, May 10-May 15, 1984, as a part of the special program on Kac-Moody Lie algebras. The purpose of the conference was to review recent developments of the theory of infinite-dimensional groups and its applications. The present collection concentrates on three very active, interrelated directions of the field: general Kac-Moody groups, gauge groups (especially loop groups) and diffeomorphism groups. I would like to express my thanks to the MSRI for sponsoring the meeting, to Ms. Faye Yeager for excellent typing, to the authors for their manuscripts, and to Springer-Verlag for publishing this volume. V. Kac INFINITE DIMENSIONAL GROUPS WITH APPLICATIONS CONTENTS The Lie Group Structure of M. Adams. T. Ratiu 1 Diffeomorphism Groups and & R. Schmid Invertible Fourier Integral Operators with Applications On Landau-Lifshitz Equation and E. Date 71 Infinite Dimensional Groups Flat Manifolds and Infinite D. S. Freed 83 Dimensional Kahler Geometry Positive-Energy Representations R. Goodman 125 of the Group of Diffeomorphisms of the Circle Instantons and Harmonic Maps M. A. Guest 137 A Coxeter Group Approach to Z. Haddad 157 Schubert Varieties Constructing Groups Associated to V. G. Kac 167 Infinite-Dimensional Lie Algebras I. Kaplansky 217 Harish-Chandra Modules Over the Virasoro Algebra & L. J. Santharoubane 233 Rational Homotopy Theory of Flag S.

kac moody algebra: Introduction to Lie Algebras K. Erdmann, Mark J. Wildon, 2006-09-28 Lie groups and Lie algebras have become essential to many parts of mathematics and theoretical physics, with Lie algebras a central object of interest in their own right. This book provides an elementary introduction to Lie algebras based on a lecture course given to fourth-year undergraduates. The only prerequisite is some linear algebra and an appendix summarizes the main facts that are needed. The treatment is kept as simple as possible with no attempt at full generality. Numerous worked examples and exercises are provided to test understanding, along with more demanding problems, several of which have solutions. Introduction to Lie Algebras covers the core material required for almost all other work in Lie theory and provides a self-study guide suitable for undergraduate students in their final year and graduate students and researchers in mathematics and theoretical physics.

kac moody algebra: Current Algebras and Groups Jouko Mickelsson, 2013-03-09 Let M be a smooth manifold and G a Lie group. In this book we shall study infinite-dimensional Lie algebras associated both to the group $\text{Map}(M, G)$ of smooth mappings from M to G and to the group of diffeomorphisms of M . In the former case the Lie algebra of the group is the algebra Mg of smooth mappings from M to the Lie algebra $\text{gof } G$. In the latter case the Lie algebra is the algebra $\text{Vect } M$ of smooth vector fields on M . However, it turns out that in many applications to field theory and statistical physics one must deal with certain extensions of the above mentioned Lie algebras. In the simplest case M is the unit circle S^1 , G is a simple finite dimensional Lie group and the central extension of $\text{Map}(S^1, \text{g})$ is an affine Kac-Moody algebra. The highest weight theory of finite dimensional Lie algebras can be extended to the case of an affine Lie algebra. The important point is that $\text{Map}(S^1, \text{g})$ can be split to positive and negative Fourier modes and the finite-dimensional piece g corresponding to the zero mode.

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focuses on: 1) Feynman integrals and modular functions, 2) hyperbolic and Lorentzian Kac-Moody algebras, related automorphic forms and applications to quantum gravity, 3) superconformal indices and elliptic hypergeometric integrals, related instanton partition functions, 4) moonshine, its arithmetic aspects, Jacobi forms, elliptic genus, and string theory, and 5) theory and applications of the elliptic Painleve equation, and aspects of Painleve equations in quantum field theories. All the topics covered are related to various partition functions emerging in different supersymmetric and ordinary quantum field theories in curved space-times of different ($d=2,3,\dots,6$) dimensions. Presenting multidisciplinary methods (localization, Borcherds products, theory of special functions, Cremona maps, etc) for treating a range of partition functions, the book is intended for graduate students and young postdocs interested in the interaction between quantum field theory and mathematics related to automorphic forms, representation theory, number theory and geometry, and mirror symmetry.

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kac moody algebra: Recent Developments in Quantum Affine Algebras and Related Topics Naihuan Jing, Kailash C. Misra, 1999 This volume reflects the proceedings of the International Conference on Representations of Affine and Quantum Affine Algebras and Their Applications held at North Carolina State University (Raleigh). In recent years, the theory of affine and quantum affine Lie algebras has become an important area of mathematical research with

numerous applications in other areas of mathematics and physics. Three areas of recent progress are the focus of this volume: affine and quantum affine algebras and their generalizations, vertex operator algebras and their representations, and applications in combinatorics and statistical mechanics. Talks given by leading international experts at the conference offered both overviews on the subjects and current research results. The book nicely presents the interplay of these topics recently occupying centre stage in the theory of infinite dimensional Lie theory.

kac moody algebra: Lie Algebras, Vertex Operator Algebras, and Related Topics Katrina Barron, Elizabeth Jurisich, Antun Milas, Kailash Misra, 2017-08-15 This volume contains the proceedings of the conference on Lie Algebras, Vertex Operator Algebras, and Related Topics, celebrating the 70th birthday of James Lepowsky and Robert Wilson, held from August 14-18, 2015, at the University of Notre Dame, Notre Dame, Indiana. Since their seminal work in the 1970s, Lepowsky and Wilson, their collaborators, their students, and those inspired by their work, have developed an amazing body of work intertwining the fields of Lie algebras, vertex algebras, number theory, theoretical physics, quantum groups, the representation theory of finite simple groups, and more. The papers presented here include recent results and descriptions of ongoing research initiatives representing the broad influence and deep connections brought about by the work of Lepowsky and Wilson and include a contribution by Yi-Zhi Huang summarizing some major open problems in these areas, in particular as they pertain to two-dimensional conformal field theory.

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