division algebra

Delving Deep into the World of Division Algebras: A Comprehensive Guide

Introduction:

Have you ever wondered about the fundamental building blocks of mathematics? Beyond the familiar realms of real and complex numbers, lies a fascinating world of algebraic structures known as division algebras. These systems, while sharing similarities with familiar number systems, possess unique properties that have captivated mathematicians for centuries. This comprehensive guide will unravel the mysteries of division algebras, exploring their definitions, characteristics, examples, and significance in various fields. We'll delve into their intricate properties, highlighting their differences and similarities with other algebraic structures. Prepare to embark on an enlightening journey into the heart of abstract algebra!

What are Division Algebras?

A division algebra is a mathematical structure that combines the properties of a field with the structure of an algebra. To break this down:

Field: A field is a set equipped with two operations, addition and multiplication, satisfying certain axioms. These axioms ensure that we can perform arithmetic operations (addition, subtraction, multiplication, and division) in a consistent and predictable way. Think of the real numbers ($\mathbb{R}$) or complex numbers ($\mathbb{C}$) – these are familiar examples of fields.

Algebra: An algebra is a vector space over a field, equipped with an additional operation of multiplication that is compatible with the vector space structure. This means that multiplication distributes over addition and is associative ($a(bc) = (ab)c$) However, unlike a field, multiplication in an algebra doesn't necessarily have to be commutative ($ab \neq ba$) or have multiplicative inverses for every non-zero element.

A division algebra, then, is an algebra where every non-zero element has a multiplicative inverse. This means that for every non-zero element 'a' in the division algebra, there exists an element 'a^{-1}' such that $a a^{-1} = a^{-1} a = 1$ (where 1 is the multiplicative identity). This property of having multiplicative inverses for all non-zero elements is crucial and distinguishes division algebras from other types of algebras.

Key Properties and Characteristics of Division Algebras:

Non-Commutativity: Unlike fields, division algebras are not necessarily commutative. This means the order of multiplication matters; a b may not equal b a. This non-commutativity introduces significant complexity and opens up a range of possibilities that don't exist in commutative fields.

Dimension: Division algebras are often defined over a specific field (like the real numbers). The dimension of a division algebra refers to its dimension as a vector space over that base field. This dimension is a crucial characteristic in classifying division algebras.

Frobenius Theorem: This fundamental theorem in algebra states that the only finite-dimensional associative division algebras over the real numbers are the real numbers ($\mathbb{R}$), the complex numbers ($\mathbb{C}$), and the quaternions ($\mathbb{H}$). This theorem elegantly limits the possibilities for associative division algebras over the real numbers.

Examples of Division Algebras:

Real Numbers ($\mathbb{R}$): The most familiar example. It's a commutative field, and thus trivially a division algebra.

Complex Numbers ($\mathbb{C}$): An extension of the real numbers, also a commutative field and a division algebra.

Quaternions ($\mathbb{H}$): A four-dimensional division algebra over the real numbers. Unlike the real and complex numbers, the quaternions are non-commutative. This non-commutativity has profound implications in various applications.

Octonions ($\mathbb{O}$): An eight-dimensional division algebra over the real numbers. They are even more exotic than quaternions, being both non-commutative and non-associative (meaning $(ab)c \neq a(bc)$ in general). Their highly non-commutative and non-associative nature makes them particularly intriguing.

Applications of Division Algebras:

Division algebras have found surprising applications in various fields:

Physics: Quaternions are used extensively in computer graphics and robotics for representing rotations in three-dimensional space. Their non-commutative nature naturally captures the complexities of three-dimensional rotations.

Engineering: The properties of quaternions and octonions find applications in signal processing and other engineering disciplines.

Mathematics: Division algebras play a crucial role in various areas of advanced mathematics, including topology, geometry, and number theory. Their abstract properties provide tools to solve problems in these domains.

The Hurwitz Theorem and its Significance:

The Hurwitz theorem provides a powerful constraint on the existence of division algebras. It states that the only finite-dimensional real normed division algebras are the real numbers, complex numbers, quaternions, and octonions. This theorem significantly restricts the possibilities for such structures. The proof of this theorem relies on sophisticated techniques from linear algebra and topology.

Further Exploration: Beyond the Basics

The study of division algebras extends far beyond the introductory concepts discussed above. Advanced topics include:

Non-associative Division Algebras: Exploring algebras where the associative property (a(bc) = (ab)c) doesn't hold. The octonions are a prime example.

Division Algebras over Other Fields: Studying division algebras defined not over the real numbers but other fields, significantly expanding the landscape of possibilities.

Algebraic K-Theory: This advanced area of mathematics uses division algebras and other algebraic structures to explore deep properties of fields and rings.

Connections to Geometry: Division algebras have intimate connections to various geometries, providing tools to analyze and understand their underlying structures.

Book Outline: "A Journey into Division Algebras"

I. Introduction:

What are division algebras?
Basic definitions and terminology
Motivation and historical context

II. Fundamental Properties:

Fields and Algebras
Associativity, Commutativity, and Distributivity
The concept of multiplicative inverses

III. Key Examples:

Real numbers (ℝ)
Complex numbers (ℂ)
Quaternions (ℍ)
Octonions (𝕆)
Comparison and contrast of properties

IV. Advanced Topics:

Frobenius Theorem
Hurwitz Theorem
Non-associative division algebras
Division algebras over other fields

V. Applications and Significance:

Applications in physics (rotations, quantum mechanics)

Applications in engineering (signal processing, robotics)
Mathematical applications (topology, geometry)

VI. Conclusion:

Summary of key concepts
Future research directions
Open problems and unanswered questions

Article Explanations (based on the book outline):

Each chapter in the book outline would delve deeply into the corresponding topics, providing rigorous mathematical explanations, examples, and proofs where appropriate. For instance, Chapter II would formally define fields and algebras, explore the axioms defining these structures, and provide clear examples to illustrate the differences between associativity, commutativity, and distributivity. Chapter III would provide a detailed examination of the real numbers, complex numbers, quaternions, and octonions, including explicit formulas for multiplication and inverses in each system. Chapter IV would introduce the Frobenius and Hurwitz Theorems, providing either proofs or detailed outlines of the proofs, alongside discussions of their significance and implications. Finally, Chapter V would explore the applications in physics, engineering, and mathematics, providing concrete examples of how division algebras are used to solve problems in these fields.

Frequently Asked Questions (FAQs):

1. What is the difference between a field and a division algebra? A field is a commutative division algebra. All fields are division algebras, but not all division algebras are fields.

1. Are quaternions commutative? No, quaternions are non-commutative; the order of multiplication matters.

1. What is the dimension of the octonions? The octonions are an 8-dimensional division algebra over the real numbers.

1. What is the significance of the Frobenius Theorem? It limits the types of finite-dimensional associative division algebras over the real numbers to only three: real numbers, complex numbers, and quaternions.

1. Are octonions associative? No, octonions are non-associative; the grouping of multiplication matters.

1. What are some applications of quaternions? Quaternions are used extensively in computer graphics and robotics for representing rotations.

1. What is the Hurwitz Theorem? It states that the only finite-dimensional real normed division algebras are the real numbers, complex numbers, quaternions, and octonions.

1. What are some open problems in the study of division algebras? Research continues on understanding division algebras over fields other than the reals, and on the classification of non-associative division algebras.

1. Why are division algebras important in mathematics? They provide powerful tools for exploring abstract algebraic structures and have surprising applications in various mathematical fields.

Related Articles:

1. Quaternion Multiplication and Rotations: Explores the use of quaternions for representing rotations in 3D space.

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1. Geometric Structures Related to Division Algebras: Discusses the relationship between division algebras and various geometric structures.

division algebra: Lectures on Division Algebras David J. Saltman, 1999 This volume is based on lectures on division algebras given at a conference held at Colorado State University. Although division algebras are a very classical object, this book presents this classical material in a new way, highlighting current approaches and new theorems, and illuminating the connections with a variety of areas in mathematics.

division algebra: *Finite-Dimensional Division Algebras over Fields* Nathan Jacobson, 2009-12-09 Here, the eminent algebraist, Nathan Jacobsen, concentrates on those algebras that have an involution. Although they appear in many contexts, these algebras first arose in the study of the so-called multiplication algebras of Riemann matrices. Of particular interest are the Jordan algebras determined by such algebras, and thus their structure is discussed in detail. Two important concepts also dealt with are the universal enveloping algebras and the reduced norm. However, the largest part of the book is the fifth chapter, which focuses on involutorial simple algebras of finite dimension over a field.

division algebra: *Moufang Sets and Structurable Division Algebras* Lien Boelaert, Tom De Medts, Anastasia Stavrova, 2019-06-10 A Moufang set is essentially a doubly transitive permutation group such that each point stabilizer contains a normal subgroup which is regular on the remaining vertices; these regular normal subgroups are called the root groups, and they are assumed to be conjugate and to generate the whole group. It has been known for some time that every Jordan division algebra gives rise to a Moufang set with abelian root groups. The authors extend this result by showing that every structurable division algebra gives rise to a Moufang set, and conversely, they show that every Moufang set arising from a simple linear algebraic group of relative rank one over an arbitrary field k of characteristic different from 2 and 3 arises from a structurable division algebra. The authors also obtain explicit formulas for the root groups, the τ-map and the Hua maps of these Moufang sets. This is particularly useful for the Moufang sets arising from exceptional linear algebraic groups.

division algebra: Division Algebras: G.M. Dixon, 2013-06-29 I don't know who Gigerenzer is, but he wrote something very clever that I saw quoted in a popular glossy magazine: Evolution has tuned the way we think to frequencies of co-occurances, as with the hunter who remembers the area where he has had the most success killing game. This sanguine thought explains my obsession with the division algebras. Every effort I have ever made to connect them to physics - to the design of reality - has succeeded, with my expectations often surpassed. Doubtless this strong statement is colored by a selective memory, but the kind of game I sought, and still seek, seems to frowst about this particular watering hole in droves. I settled down there some years ago and have never feIt like Ieaving. This book is about the beasts I selected for attention (if you will, to ren der this metaphor politically correct, let's say I was a nature photographer), and the kind of tools I had to develop to get the kind of shots Iwanted (the tools that I found there were for my taste overly abstract and theoretical). Half of thisbook is about these tools, and some applications thereof that should demonstrate their power. The rest is devoted to a demonstration of the intimate connection between the mathematics of the division algebras and the Standard Model of quarks and leptons with U(l) x SU(2) x SU(3) gauge fields, and the connection of this model to lO-dimensional spacetime implied by the mathematics.

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division algebra: Cyclic Division Algebras Frdrique Oggier, Jean-Claude Belfiore, Emanuele Viterbo, 2007 Multiple antennas at both the transmitter and receiver ends of a wireless digital transmission channel may increase both data rate and reliability. Reliable high rate transmission over such channels can only be achieved through Space-Time coding. Rank and determinant code design criteria have been proposed to enhance diversity and coding gain. The special case of full-diversity criterion, requires that the difference of any two distinct codewords has full rank. Extensive work has been done on Space-Time coding, aiming to attain fully diverse codes with high rate. Division algebras have been proposed as a new tool for constructing Space-Time codes, since they are non-commutative algebras that naturally yield linear fully diverse codes. Their algebraic properties can thus be further exploited to improve the design of good codes. Cyclic Division Algebras: A Tool for Space-Time Coding provides a tutorial introduction to the algebraic tools involved in the design of codes based on division algebras. The different design criteria involved are illustrated, including the constellation shaping, the information lossless property, the non-vanishing determinant property and the diversity multiplexing tradeoff. Finally complete mathematical background underlying the construction of the Golden code and the other Perfect Space-Time block codes is given. Cyclic Division Algebras: A Tool for Space-Time Coding is for students, researchers and professionals working on wireless communication systems.

division algebra: *On The Role Of Division, Jordan And Related Algebras In Particle Physics* Feza Gursey, Chia-hsiung Tze, 1996-11-22 This monograph surveys the role of some associative and non-associative algebras, remarkable by their ubiquitous appearance in contemporary theoretical physics, particularly in particle physics. It concerns the interplay between division algebras, specifically quaternions and octonions, between Jordan and related algebras on the one hand, and unified theories of the basic interactions on the other. Selected applications of these algebraic structures are discussed: quaternion analyticity of Yang-Mills instantons, octonionic aspects of exceptional broken gauge, supergravity theories, division algebras in anyonic phenomena and in theories of extended objects in critical dimensions. The topics presented deal primarily with original contributions by the authors.

division algebra: Linear Algebra Over Division Ring Aleks Kleyn, 2014-10-27 In this book I treat linear maps of vector space over division ring. The set of linear maps of left vector space over division ring D is right vector space over division ring D. The concept of twin representations follows from the joint consideration of vector space V and vector space of linear transformations of the vector space V. Considering of twin representations of division ring in Abelian group leads to the concept of D-vector space and their linear map. Based on polylinear map I considered definition of tensor product of rings and tensor product of D-vector spaces.

division algebra: Structure of Algebras Abraham Adrian Albert, 1939-12-31 The first three chapters of this work contain an exposition of the Wedderburn structure theorems. Chapter IV contains the theory of the commutator subalgebra of a simple subalgebra of a normal simple algebra, the study of automorphisms of a simple algebra, splitting fields, and the index reduction factor theory. The fifth chapter contains the foundation of the theory of crossed products and of their special case, cyclic algebras. The theory of exponents is derived there as well as the consequent factorization of normal division algebras into direct factors of prime-power degree. Chapter VI consists of the study of the abelian group of cyclic systems which is applied in Chapter VII to yield the theory of the structure of direct products of cyclic algebras and the consequent properties of norms in cyclic fields. This chapter is closed with the theory of p-algebras. In Chapter VIII an exposition is given of the theory of the representations of algebras. The treatment is somewhat novel in that while the recent expositions have used representation theorems to obtain a number of results on algebras, here the theorems on algebras are themselves used in the derivation of results on representations. The presentation has its inspiration in the author's work on the theory of Riemann matrices and is concluded by the introduction to the generalization (by H. Weyl and the author) of that theory. The theory of involutorial simple algebras is derived in Chapter X both for algebras over general fields and over the rational field. The results are also applied in the determination of the structure of the multiplication algebras of all generalized Riemann matrices, a result which is seen in Chapter XI to imply a complete solution of the principal problem on Riemann matrices.

division algebra: Lectures on Division Algebras David J. Saltman, This volume is based on lectures on division algebras given at a conference held at Colorado State University. Although division algebras are a very classical object, this book presents this classical material in a new way, highlighting current approaches and new theorems, and illuminating the connections with a variety of areas in mathematics.

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division algebra: The Concise Handbook of Algebra Alexander V. Mikhalev, G.F. Pilz, 2013-06-29 It is by no means clear what comprises the heart or core of algebra, the part of algebra which every algebraist should know. Hence we feel that a book on our heart might be useful. We have tried to catch this heart in a collection of about 150 short sections, written by leading algebraists in these areas. These sections are organized in 9 chapters A, B, . . . , I. Of course, the selection is partly based on personal preferences, and we ask you for your understanding if some selections do not meet your taste (for unknown reasons, we only had problems in the chapter Groups to get enough articles in time). We hope that this book sets up a standard of what all algebraists are supposed to know in their chapters; interested people from other areas should be able to get a quick idea about the area. So the target group consists of anyone interested in algebra, from graduate students to established researchers, including those who want to obtain a quick overview or a better understanding of our selected topics. The prerequisites are something like the contents of standard textbooks on higher algebra. This book should also enable the reader to read the big Handbook (Hazewinkel 1999-) and other handbooks. In case of multiple authors, the authors are listed alphabetically; so their order has nothing to do with the amounts of their contributions.

division algebra: *A Book of Abstract Algebra* Charles C Pinter, 2010-01-14 Accessible but rigorous, this outstanding text encompasses all of the topics covered by a typical course in elementary abstract algebra. Its easy-to-read treatment offers an intuitive approach, featuring informal discussions followed by thematically arranged exercises. This second edition features additional exercises to improve student familiarity with applications. 1990 edition.

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division algebra: Collected Mathematical Papers: Associative algebras and Riemann matrices Abraham Adrian Albert, Richard E. Block, This book contains the collected works of A. Adrian Albert, a leading algebraist of the twentieth century. Albert made many important contributions to the theory of the Brauer group and central simple algeras, Riemann matrices, nonassociative algebras and other topics. Part 1 focuses on associative algebras and Riemann matrices part 2 on nonassociative algebras and miscellany. Because much of Albert's work remains of vital interest in contemporary research, this volume will interst mathematicians in a variety of areas.

division algebra: Introduction to Noncommutative Algebra Matej Brešar, 2014-10-14 Providing an elementary introduction to noncommutative rings and algebras, this textbook begins with the classical theory of finite dimensional algebras. Only after this, modules, vector spaces over division rings, and tensor products are introduced and studied. This is followed by Jacobson's structure theory of rings. The final chapters treat free algebras, polynomial identities, and rings of quotients. Many of the results are not presented in their full generality. Rather, the emphasis is on clarity of exposition and simplicity of the proofs, with several being different from those in other texts on the subject. Prerequisites are kept to a minimum, and new concepts are introduced gradually and are carefully motivated. Introduction to Noncommutative Algebra is therefore accessible to a wide mathematical audience. It is, however, primarily intended for beginning graduate and advanced undergraduate students encountering noncommutative algebra for the first time.

division algebra: Value Functions on Simple Algebras, and Associated Graded Rings Jean-Pierre Tignol, Adrian R. Wadsworth, 2015-04-03 This monograph is the first book-length treatment of valuation theory on finite-dimensional division algebras, a subject of active and substantial research over the last forty years. Its development was spurred in the last decades of the twentieth century by important advances such as Amitsur's construction of non crossed products and Platonov's solution of the Tannaka-Artin problem. This study is particularly timely because it approaches the subject from the perspective of associated graded structures. This new approach has been developed by the authors in the last few years and has significantly clarified the theory. Various constructions of division algebras are obtained as applications of the theory, such as noncrossed products and indecomposable algebras. In addition, the use of valuation theory in reduced Whitehead group calculations (after Hazrat and Wadsworth) and in essential dimension computations (after Baek and Merkurjev) is showcased. The intended audience consists of graduate students and research mathematicians.

division algebra: *Mathematics For Physics: An Illustrated Handbook* Adam Marsh, 2017-11-27 This unique book complements traditional textbooks by providing a visual yet rigorous survey of the mathematics used in theoretical physics beyond that typically covered in undergraduate math and physics courses. The exposition is pedagogical but compact, and the emphasis is on defining and visualizing concepts and relationships between them, as well as listing common confusions, alternative notations and jargon, and relevant facts and theorems. Special attention is given to detailed figures and geometric viewpoints. Certain topics which are well covered in textbooks, such as historical motivations, proofs and derivations, and tools for practical calculations, are avoided. The primary physical models targeted are general relativity, spinors, and gauge theories, with notable chapters on Riemannian geometry, Clifford algebras, and fiber bundles.

division algebra: Quaternion Algebras John Voight, 2021-06-28 This open access textbook presents a comprehensive treatment of the arithmetic theory of quaternion algebras and orders, a subject with applications in diverse areas of mathematics. Written to be accessible and approachable to the graduate student reader, this text collects and synthesizes results from across the literature. Numerous pathways offer explorations in many different directions, while the unified treatment makes this book an essential reference for students and researchers alike. Divided into five parts, the book begins with a basic introduction to the noncommutative algebra underlying the theory of quaternion algebras over fields, including the relationship to quadratic forms. An in-depth exploration of the arithmetic of quaternion algebras and orders follows. The third part considers analytic aspects, starting with zeta functions and then passing to an idelic approach, offering a pathway from local to global that includes strong approximation. Applications of unit groups of quaternion orders to hyperbolic geometry and low-dimensional topology follow, relating geometric and topological properties to arithmetic invariants. Arithmetic geometry completes the volume, including quaternionic aspects of modular forms, supersingular elliptic curves, and the moduli of QM abelian surfaces. Quaternion Algebras encompasses a vast wealth of knowledge at the intersection of many fields. Graduate students interested in algebra, geometry, and number theory will appreciate the many avenues and connections to be explored. Instructors will find numerous options for constructing introductory and advanced courses, while researchers will value the all-embracing treatment. Readers are assumed to have some familiarity with algebraic number theory and commutative algebra, as well as the fundamentals of linear algebra, topology, and complex analysis. More advanced topics call upon additional background, as noted, though essential concepts and motivation are recapped throughout.

division algebra: Associative Algebras R.S. Pierce, 2012-12-06 For many people there is life after 40; for some mathematicians there is algebra after Galois theory. The objective ofthis book is to prove the latter thesis. It is written primarily for students who have assimilated substantial portions of a standard first year graduate algebra textbook, and who have enjoyed the experience. The material that is presented here should not be fatal if it is swallowed by persons who are not members of that group. The objects of our attention in this book are associative algebras, mostly the ones that are finite dimensional over a field. This subject is ideal for a textbook that will lead graduate students into a specialized field of research. The major theorems on associative algebras inc1ude some of the most splendid results of the great heros of algebra: Wedderbum, Artin, Noether, Hasse, Brauer, Albert, Jacobson, and many others. The process of refine ment and c1arification has brought the proof of the gems in this subject to a level that can be appreciated by students with only modest background. The subject is almost unique in the wide range of contacts that it makes with other parts of mathematics. The study of associative algebras con tributes to and draws from such topics as group theory, commutative ring theory, field theory, algebraic number theory, algebraic geometry, homo logical algebra, and category theory. It even has some ties with parts of applied mathematics.

division algebra: Algebra Thomas W. Hungerford, 2003-02-14 Finally a self-contained, one volume, graduate-level algebra text that is readable by the average graduate student and flexible enough to accommodate a wide variety of instructors and course contents. The guiding principle throughout is that the material should be presented as general as possible, consistent with good pedagogy. Therefore it stresses clarity rather than brevity and contains an extraordinarily large number of illustrative exercises.

division algebra: Azumaya Algebras, Actions, and Modules Darrell Haile, 1992 This volume contains the proceedings of a conference in honor of Goro Azumaya's seventieth birthday, held at Indiana University of Bloomington in May 1990. Professor Azumaya, who has been on the faculty of Indiana University since 1968, has made many important contributions to modern abstract algebra. His introduction and investigation of what have come to be known as Azumaya algebras subsequently stimulated much research on such rings and algebras, as well as applications to geometry and number theory. In addition to honoring Professor Azumaya's contributions, the conference was intended to stimulate interaction among three areas of his research interests; Azumaya algebras, group and Hopf algebra actions, and module theory. Aimed at researchers in algebra, this volume contains contributions by some of the leaders in these areas.

division algebra: An Introduction to Nonassociative Algebras Richard Donald Schafer, 2017-12-13 An important addition to the mathematical literature … contains very interesting results not available in other books; written in a plain and clear style, it reads very smoothly. — Bulletin of the American Mathematical Society This concise study was the first book to bring together material on the theory of nonassociative algebras, which had previously been scattered throughout the literature. It emphasizes algebras that are, for the most part, finite-dimensional over a field. Written as an introduction for graduate students and other mathematicians meeting the subject for the first time, the treatment's prerequisites include an acquaintance with the fundamentals of abstract and linear algebra. After an introductory chapter, the book explores arbitrary nonassociative algebras and alternative algebras. Subsequent chapters concentrate on Jordan algebras and power-associative algebras. Throughout, an effort has been made to present the basic ideas, techniques, and flavor of what happens when the associative law is not assumed. Many of the proofs are given in complete detail.

division algebra: Introduction to Octonion and Other Non-Associative Algebras in Physics Susumu Okubo, 1995-08-03 In this book, the author aims to familiarize researchers and graduate students in both physics and mathematics with the application of non-associative algebras in physics.Topics covered by the author range from algebras of observables in quantum mechanics, angular momentum and octonions, division algebra, triple-linear products and YangSHBaxter equations. The author also covers non-associative gauge theoretic reformulation of Einstein's general relativity theory and so on. Much of the material found in this book is not available in other standard works.

division algebra: Introduction to Quadratic Forms over Fields T.Y. Lam, This new version of the author's prizewinning book, Algebraic Theory of Quadratic Forms (W. A. Benjamin, Inc., 1973), gives a modern and self-contained introduction to the theory of quadratic forms over fields of characteristic different from two. Starting with few prerequisites beyond linear algebra, the author charts an expert course from Witt's classical theory of quadratic forms, quaternion and Clifford algebras, Artin-Schreier theory of formally real fields, and structural theorems on Witt rings, to the theory of Pfister forms, function fields, and field invariants. These main developments are seamlessly interwoven with excursions into Brauer-Wall groups, local and global fields, trace forms, Galois theory, and elementary algebraic K-theory, to create a uniquely original treatment of quadratic form theory over fields. Two new chapters totaling more than 100 pages have been added to the earlier incarnation of this book to take into account some of the newer results and more recent viewpoints in the area. As is characteristic of this author's expository style, the presentation of the main material in this book is interspersed with a copious number of carefully chosen examples to illustrate the general theory. This feature, together with a rich stock of some 280 exercises for the thirteen chapters, greatly enhances the pedagogical value of this book, both as a graduate text and as a reference work for researchers in algebra, number theory, algebraic geometry, algebraic topology, and geometric topology.

division algebra: Algebras and Modules II Idun Reiten, Sverre O. Smalø, Øyvind Solberg, Canadian Mathematical Society, 1998 The 43 research papers demonstrate the application of recent developments in the representation theory of artin algebras and related topics. Among the algebras considered are tame, bi- serial, cellular, factorial hereditary, Hopf, Koszul, non- polynomial growth, pre-projective, Termperley-Lieb, tilted, and quasi-tilted. Other topics include tilting and co-tilting modules and generalizations as *-modules, exceptional sequences of modules and vector bundles, homological conjectives, and vector space categories. The treatment assumes knowledge of non-commutative algebra, including rings, modules, and homological algebra at a graduate or professional level. No index. Member prices are $79 for institutions and $59 for individuals, which also apply to members of the Canadian Mathematical Society. Annotation copyrighted by Book News, Inc., Portland, OR

division algebra: Let's Play Math Denise Gaskins, 2012-09-04

division algebra: Topological Geometry Ian R. Porteous, 1981-02-05 The earlier chapter of this self-contained text provide a route from first principles through standard linear and quadratic algebra to geometric algebra, with Clifford's geometric algebras taking pride of place. In parallel with this is an account, also from first principles, of the elementary theory of topological spaces and of continuous and differentiable maps that leads up to the definitions of smooth manifolds and their tangent spaces and of Lie groups and Lie algebras. The calculus is presented as far as possible in basis free form to emphasize its geometrical flavour and its linear algebra content. In this second edition Dr Porteous has taken the opportunity to add a chapter on triality which extends earlier work on the Spin groups in the chapter on Clifford algebras. The details include a number of important transitive group actions and a description of one of the exceptional Lie groups, the group G2. A number of corrections and improvements have also been made. There are many exercises throughout the book and senior undergraduates in mathematics as well as first-year graduate students will continue to find it stimulating and rewarding.

division algebra: *Central Simple Algebras and Galois Cohomology* Philippe Gille, Tamás Szamuely, 2017-08-10 The first comprehensive, modern introduction to the theory of central simple algebras over arbitrary fields, this book starts from the basics and reaches such advanced results as the Merkurjev–Suslin theorem, a culmination of work initiated by Brauer, Noether, Hasse and Albert, and the starting point of current research in motivic cohomology theory by Voevodsky, Suslin, Rost and others. Assuming only a solid background in algebra, the text covers the basic theory of central simple algebras, methods of Galois descent and Galois cohomology, Severi–Brauer varieties, and techniques in Milnor K-theory and K-cohomology, leading to a full proof of the Merkurjev–Suslin theorem and its application to the characterization of reduced norms. The final chapter rounds off the theory by presenting the results in positive characteristic, including the theorems of Bloch–Gabber–Kato and Izhboldin. This second edition has been carefully revised and updated, and contains important additional topics.

division algebra: *Structure and Representations of Jordan Algebras* Nathan Jacobson, 1968-12-31 The theory of Jordan algebras has played important roles behind the scenes of several areas of mathematics. Jacobson's book has long been the definitive treatment of the subject. It covers foundational material, structure theory, and representation theory for Jordan algebras. Of course, there are immediate connections with Lie algebras, which Jacobson details in Chapter 8. Of particular continuing interest is the discussion of exceptional Jordan algebras, which serve to explain the exceptional Lie algebras and Lie groups. Jordan algebras originally arose in the attempts by Jordan, von Neumann, and Wigner to formulate the foundations of quantum mechanics. They are still useful and important in modern mathematical physics, as well as in Lie theory, geometry, and certain areas of analysis.

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